MCM 2015

Applications and Some New Result on Compute and Forward

Giuseppe Caire

Technische Universität Berlin

Technische Universität Berlin

Alexander von Humboldt Stiftung/Foundation

July 30-31, 2015, Munich, Germany

CommIT Communications and Information Theory Chair

Outline

- Families of Nested Lattice Codes.
- Compute and Forward and with Unequal Powers and Rates
- Application to the MIMO MAC and MIMO compound MAC.
- Application to DAS: MAC-Relay (uplink) and BC-Relay (downlink)
- Application to Network-Coded CIC
- Application to Cascade of Alternating HD Relays
- Compute and Forward with Successive Decoding

Acknowledgements

Joint work with:

- Song-Nam Hong, Vasilis Ntranos (USC)
- Bobak Nazer (Boston University)
- Vivek Cadambe (Pennsylvania State University)

Lattices

- A Lattice is a $\mathbb{Z}$-module embedded in $\mathbb{R}^n$:

$$\Lambda = \{\underline{\boldsymbol{\lambda}} = \underline{\mathbf{z}}\mathbf{M} : \underline{\mathbf{z}} \in \mathbb{Z}^n\}$$

where $\mathbf{M} \in \mathbb{R}^{n \times n}$ is a full-rank generator matrix.

- Λ is an additive group.

- For $\underline{\mathbf{r}} \in \mathbb{R}^n$, we define the lattice quantization function:

$$Q_\Lambda(\underline{\mathbf{r}}) = \text{argmin}_{\underline{\boldsymbol{\lambda}} \in \Lambda} \|\underline{\mathbf{r}} - \underline{\boldsymbol{\lambda}}\|^2$$

- The Voronoi region (fundamental cell) of Λ is:

$$\mathcal{V}_\Lambda = \{\underline{\mathbf{r}} \in \mathbb{R}^n : Q_\Lambda(\underline{\mathbf{r}}) = \underline{\mathbf{0}}\}$$

Lattices

- Modulo-Λ reduction

$$[\underline{\mathbf{r}}] \mod \Lambda = \underline{\mathbf{r}} - Q_\Lambda(\underline{\mathbf{r}})$$

- Per-component second moment

$$\sigma_\Lambda^2 = \frac{1}{n\mathsf{Vol}(\mathcal{V})} \int_{\mathcal{V}} \|\underline{\mathbf{r}}\|^2 d\underline{\mathbf{r}}.$$

CommIT Communications and Information Theory Chair

Lattices and linear codes

- Consider a $k_F \times n$ matrix $\mathbf{G}$ over $\mathbb{Z}_p$, where p is a prime.
- For $\ell = 1, \ldots, L$, and $k_C \leq k_{C,\ell} < k_{F,\ell} \leq k_F$, let $\mathbf{G}_{F,\ell}$ and $\mathbf{G}_{C,\ell}$ be the submatrices formed by the first $k_{F,\ell}$ and $k_{C,\ell}$ rows of $\mathbf{G}$, respectively.
- Let $\mathcal{C}_{F,\ell}$ and $\mathcal{C}_{C,\ell}$ denote the row spaces of $\mathbf{G}_{F,\ell}$ and $\mathbf{G}_{C,\ell}$, respectively.
- Define the mappings $\phi : \mathbb{Z}_p \to \mathbb{R}$ and $\bar{\phi} : \gamma p^{-1}\mathbb{Z} \to \mathbb{Z}_p$ such that

$$\phi(w) = \gamma p^{-1} w, \qquad \bar{\phi}(\kappa) = \left[\gamma^{-1} p \kappa\right] \mod p$$

- Following a scaled version of Construction A, we create the lattices

$$\begin{aligned} \Lambda_{C,\ell} &= \left\{\underline{\boldsymbol{\lambda}} \in \gamma p^{-1}\mathbb{Z}^n : \bar{\phi}(\underline{\boldsymbol{\lambda}}) \in \mathcal{C}_{C,\ell}\right\} \\ \Lambda_{F,\ell} &= \left\{\underline{\boldsymbol{\lambda}} \in \gamma p^{-1}\mathbb{Z}^n : \bar{\phi}(\underline{\boldsymbol{\lambda}}) \in \mathcal{C}_{F,\ell}\right\} \end{aligned}$$

Lattices and linear codes

- By construction, for all $\ell = 1, \dots, L$, we have

$$\Lambda_C \subseteq \Lambda_{C,\ell} \subset \Lambda_{F,\ell} \subseteq \Lambda_F$$

- The ℓ-th nested lattice code of the family is given by

$$\mathcal{L}_\ell = \Lambda_{F,\ell} \cap \mathcal{V}_{\Lambda_{C,\ell}}$$

and has rate (bit per symbol)

$$R_\ell = \frac{k_{F,\ell} - k_{C,\ell}}{n} \log p$$

Coding performance

- Choose power levels $P_\ell : \ell = 1, \ldots, L$ and let $P_{\text{max}} = \max_\ell P_\ell$ and V_n be the volume of an n-dimensional ball of radius 1.
- Choose noise tolerances $\sigma^2_{\text{eff},\ell} : \ell = 1, \ldots, L$.
- Set p to be the largest prime between $\frac{1}{2}n^{3/2}$ and $n^{3/2}$, which is guaranteed to exist for $n > 3$ by Bertrand's Postulate, and let

$$\gamma = 2\sqrt{nP_{\text{max}}}$$

$$k_{C,\ell} = \frac{n}{2\log p}\left(\log\left(\frac{P_{\text{max}}}{P_\ell - \mu}\right) + \log\left(\frac{4}{V_n^{2/n}}\right) + \delta_C\right)$$

$$k_{F,\ell} = \frac{n}{2\log p}\left(\log\left(\frac{\gamma^2}{2\pi e \sigma^2_{\text{eff},\ell}}\right) - \delta_F\right)$$

where $\mu, \delta_C, \delta_F > 0$.

Coding performance

Theorem 1:

Choose $P_\ell > 0$ and effective noise tolerances $0 < \sigma^2_{\text{eff},\ell} < P_\ell : \ell = 1, \ldots, L$. For any $\epsilon > 0$ and n large enough, there are constants $\delta_C, \delta_F > 0$ such that for the choices of $k_{C,\ell}, k_{F,\ell}$ there exists a matrix $\mathbf{G} \in \mathbb{Z}_p^{k_F \times n}$, such that, for all $\ell = 1, \ldots, L$,

(a) the submatrices $\mathbf{G}_{C,\ell}, \mathbf{G}_{F,\ell}$ are full rank.

(b) the coarse lattices $\Lambda_{C,\ell}$ have second moments close to the power constraint

$$P_\ell - \epsilon < \sigma^2(\Lambda_{C,\ell}) \leq P_\ell \ .$$

(c) the fine lattices $\Lambda_{F,\ell}$ tolerate the prescribed level of effective noise. Specifically, consider any linear mixture of Gaussian and Voronoi-shaped noise of the form $\underline{\mathbf{z}}_{\text{eff}} = \beta_0 \underline{\mathbf{z}}_0 + \sum_{\ell=1}^{L} \beta_\ell \underline{\mathbf{z}}_\ell$ where $\beta_0, \beta_1, \ldots, \beta_L \in \mathbb{R}$, $\underline{\mathbf{z}}_0 \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$, $\underline{\mathbf{z}}_\ell \sim \text{Unif}(\mathcal{V}_{C,\ell})$, and the noise components $\underline{\mathbf{z}}_0, \ldots, \underline{\mathbf{z}}_L$ are independent of each other and $\underline{\boldsymbol{\lambda}}$. Then, for any fine lattice point $\underline{\boldsymbol{\lambda}} \in \Lambda_{F,\ell}$ and any coarse lattice $\Lambda_{C,m} \subset \Lambda_{F,\ell}$,

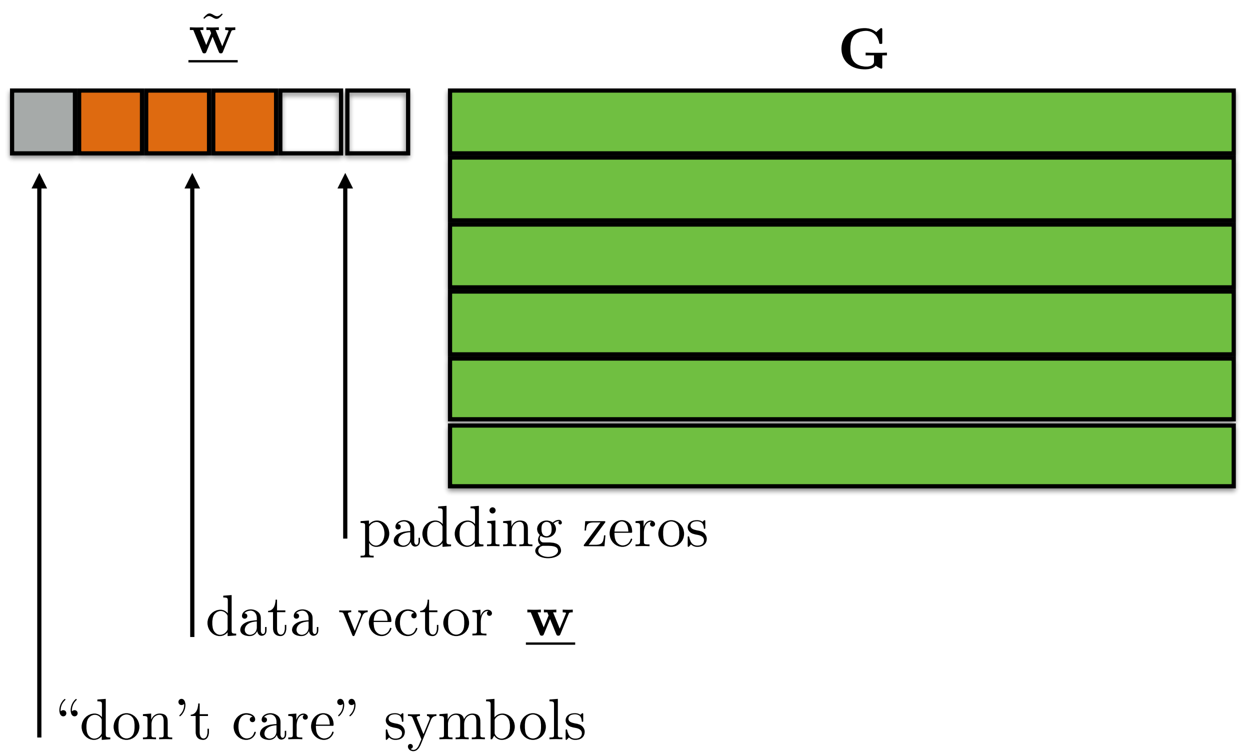
$$\mathbb{P}\left(\left[Q_{\Lambda_{F,\ell}}(\underline{\boldsymbol{\lambda}} + \underline{\mathbf{z}}_{\text{eff}})\right] \mod \Lambda_{C,m} \neq [\underline{\boldsymbol{\lambda}}] \mod \Lambda_{C,m}\right) < \epsilon$$

if $\beta_0^2 + \sum_{\ell=1}^{L} \beta_\ell^2 P_\ell \leq \sigma_{\text{eff},\ell}^2$.

(d) the nested lattice codebooks $\mathcal{L}_\ell = \Lambda_{F,\ell} \cap \mathcal{V}_{C,\ell}$ have appropriate rates

$$\frac{1}{n} \log \left|\mathcal{L}_\ell\right| = \frac{k_{F,\ell} - k_{C,\ell}}{n} \log p > \frac{1}{2} \log\left(\frac{P_\ell}{\sigma_{\text{eff},\ell}^2}\right) - \epsilon \; .$$

"Signal levels" interpretation



Nazer and Gastpar Compute and Forward (CoF)

- L-user Gaussian MAC:

$$\underline{\mathbf{y}} = \sum_{\ell=1}^{L} h_\ell \underline{\mathbf{x}}_\ell + \underline{\mathbf{z}} = \mathbf{h}^{\mathsf{T}} \underline{\mathbf{X}} + \underline{\mathbf{z}}$$

- Each user ℓ makes use of the same nested lattice code $\mathcal{L} = \Lambda_F \cap \mathcal{V}_{\Lambda_C}$ and transmits with dithering.

- Goal: use lattice decoding to decode an integer linear combination

$$\underline{\mathbf{s}} = \sum_{\ell=1}^{L} a_\ell \underline{\mathbf{t}}_\ell = \mathbf{a}^{\mathsf{T}} \underline{\mathbf{T}}$$

- Lattice linearity yields that

$$\underline{\mathbf{u}} = \bigoplus_{\ell=1}^{L} q_\ell \underline{\mathbf{w}}_\ell$$

with $\mathbf{q} = [\mathbf{a}] \mod p$.

- Computation rate: maximum rate $\frac{k_F - k_C}{n} \log p$ (in bit/read dimension) for which the desired linear combination can be decoded with vanishing probability of error.

- The receiver computes

$$\begin{aligned} \underline{\mathbf{y}}' &= [\alpha \underline{\mathbf{y}} - \mathbf{a}^{\mathsf{T}} \underline{\mathbf{D}}] \mod \Lambda \\ &= \left[\mathbf{a}^{\mathsf{T}} \underline{\mathbf{T}} + (\alpha \mathbf{h} - \mathbf{a})^{\mathsf{T}} \underline{\mathbf{X}} + \alpha \underline{\mathbf{z}}\right] \mod \Lambda \end{aligned}$$

- Applying lattice decoding, we get the achievable computation rate

$$R_{\text{comp}}(P, \sigma^2_{\text{eff}}) = \frac{1}{2} \log^+ \left(\frac{P}{\sigma^2_{\text{eff}}} \right)$$

- Effective noise

$$\underline{\mathbf{z}}_{\text{eff}} = (\alpha \mathbf{h} - \mathbf{a})^{\mathsf{T}} \underline{\mathbf{X}} + \alpha \underline{\mathbf{z}}$$

with

$$\sigma^2_{\text{eff}} = \|\alpha \mathbf{h} - \mathbf{a}\|^2 P + |\alpha|^2$$

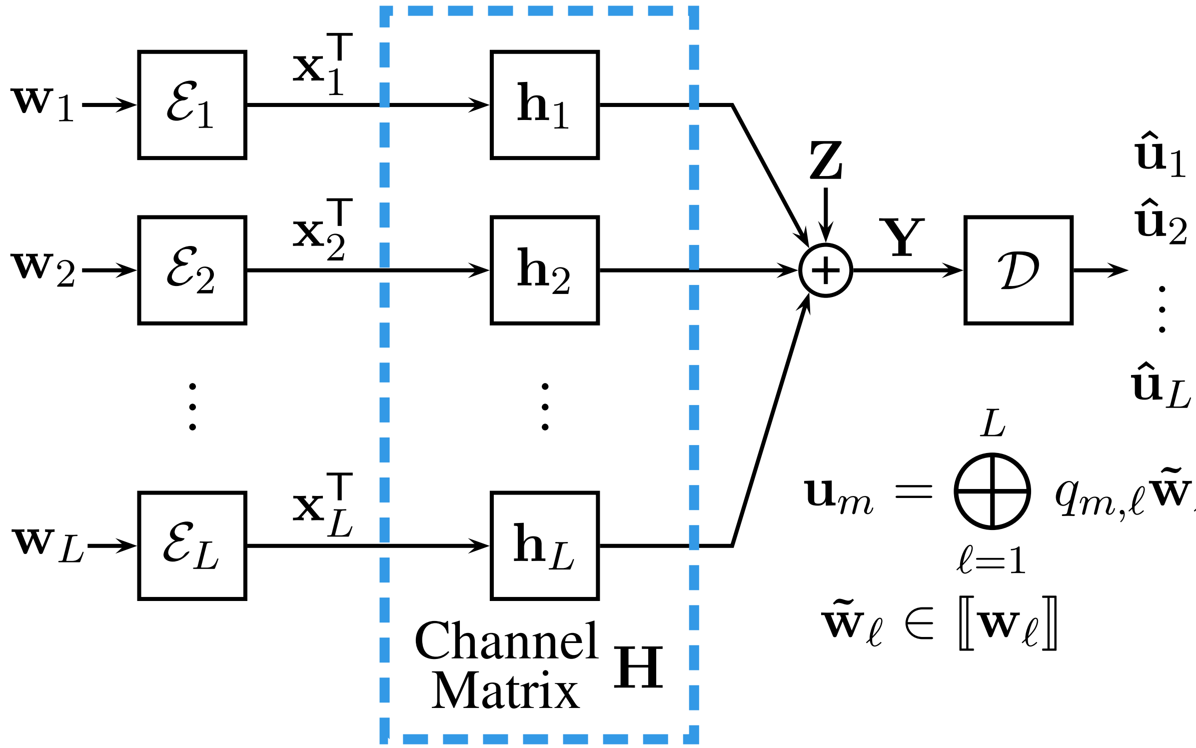
Communications and Information Theory Chair

- Minimizing with respect to α, we find

$$\sigma_{\text{eff}}^2 = \mathbf{a}^{\mathsf{H}} \left(P^{-1}\mathbf{I} + \mathbf{h}\mathbf{h}^{\mathsf{H}} \right)^{-1} \mathbf{a}$$

which can be minimized by using Lattice reduction (LLL, Minkowsky, Phost "sphere" decoding) with respect to $\mathbf{a} \in \mathbb{Z}^K[j]$.

MIMO receiver



$$\mathbf{u}_m = \bigoplus_{\ell=1}^{L} q_{m,\ell} \tilde{\mathbf{w}}_\ell$$

$$\tilde{\mathbf{w}}_\ell \in [\![\mathbf{w}_\ell]\!]$$

MIMO receiver

- The channel model becomes

$$\underline{\mathbf{Y}} = \mathbf{H}\underline{\mathbf{X}} + \underline{\mathbf{Z}}$$

- Apply the equalization vector $\mathbf{b} \in \mathbb{R}^{N_\mathsf{r}}$, and obtain

$$\mathbf{b}^\mathsf{H}\underline{\mathbf{Y}} = \sum_{\ell=1}^{L} a_\ell \underline{\mathbf{t}}_\ell + \underbrace{\sum_{\ell=1}^{L} \left(\mathbf{b}^\mathsf{H}\mathbf{h}_\ell - a_\ell\right)\underline{\mathbf{x}}_\ell + \mathbf{b}^\mathsf{H}\underline{\mathbf{Z}}}_{\text{effective noise}}$$

- The achievable computation rate is

$$R_{\text{comp}}(\mathbf{H}, \mathbf{a}) = \frac{1}{2}\log^+\left(\frac{P}{\mathbf{a}^\mathsf{H}\left(P^{-1}\mathbf{I} + \mathbf{H}^\mathsf{H}\mathbf{H}\right)^{-1}\mathbf{a}}\right)$$

Multiple equations and unequal power and rates

- We can generalize the above construction by considering a family of nested lattice codes with different powers P_ℓ and different rates R_ℓ as described above.

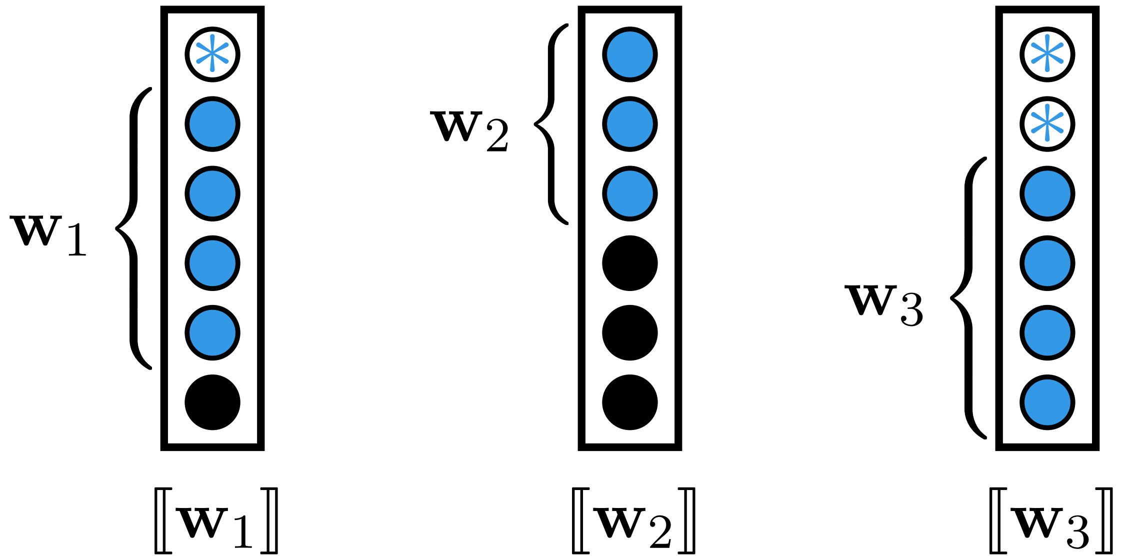
- In this case, we aim at recovering an integer combination of the message cosets (arbitrary “don’t care symbols”, and zero padding symbols).

$$\underline{\mathbf{u}}_m = \bigoplus_{\ell=1}^{L} q_{m,\ell} \tilde{\underline{\mathbf{w}}}_\ell$$

where $q_{m,\ell} \in \mathbb{Z}_p$ and $\tilde{\underline{\mathbf{w}}}_\ell$ is an element of a certain coset $[[\underline{\mathbf{w}}_\ell]]$ with respect to the message $\underline{\mathbf{w}}_\ell$.

- Furthermore, we can obtain up to L such linear combinations, according to an integer-valued matrix $\mathbf{A} \in \mathbb{Z}^{L \times L}$.

Message cosets



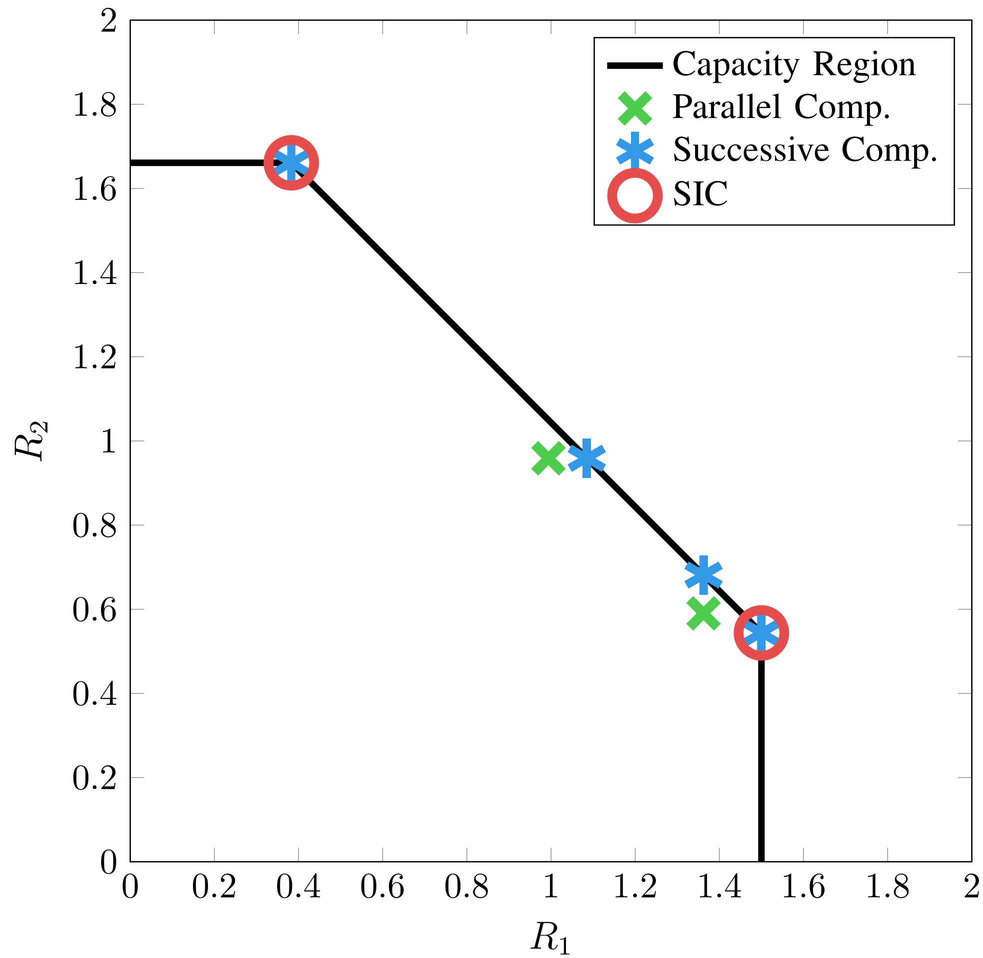
Computation rate region

- $\mathcal{R}_{\text{comp}}(\mathbf{H}, \mathbf{A})$ is the closure of the subset of $\mathbb{R}_+^L$ of all rate points $(R_1, \ldots, R_L)$ for which there exist codes with vanishing probability of error of all L linear combinations.

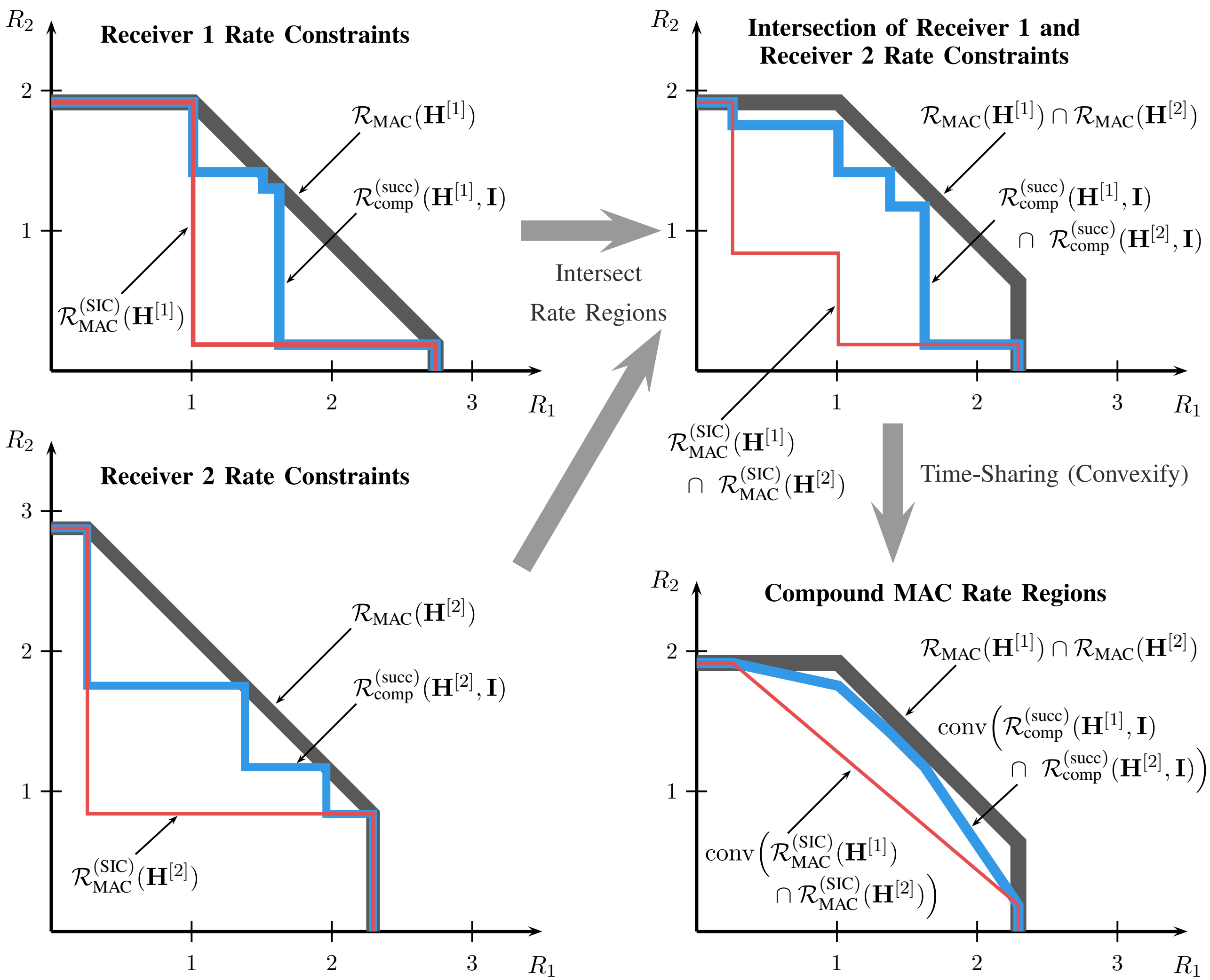
- To this regard, we have:

1. Achievable $\mathcal{R}_{\text{comp}}(\mathbf{H}, \mathbf{A})$ with parallel computation.
2. Achievable $\mathcal{R}_{\text{comp}}(\mathbf{H}, \mathbf{A})$ with successive computation.
3. Multiple-access sum capacity within L bits: the sum of the L largest parallel computation rates is larger or equal to the MAC sum capacity minus L bits.
4. Exact multiple-access sum capacity: for any unimodular $\mathbf{A}$ the sum of the L successive computation rates is equal to the MAC sum capacity.

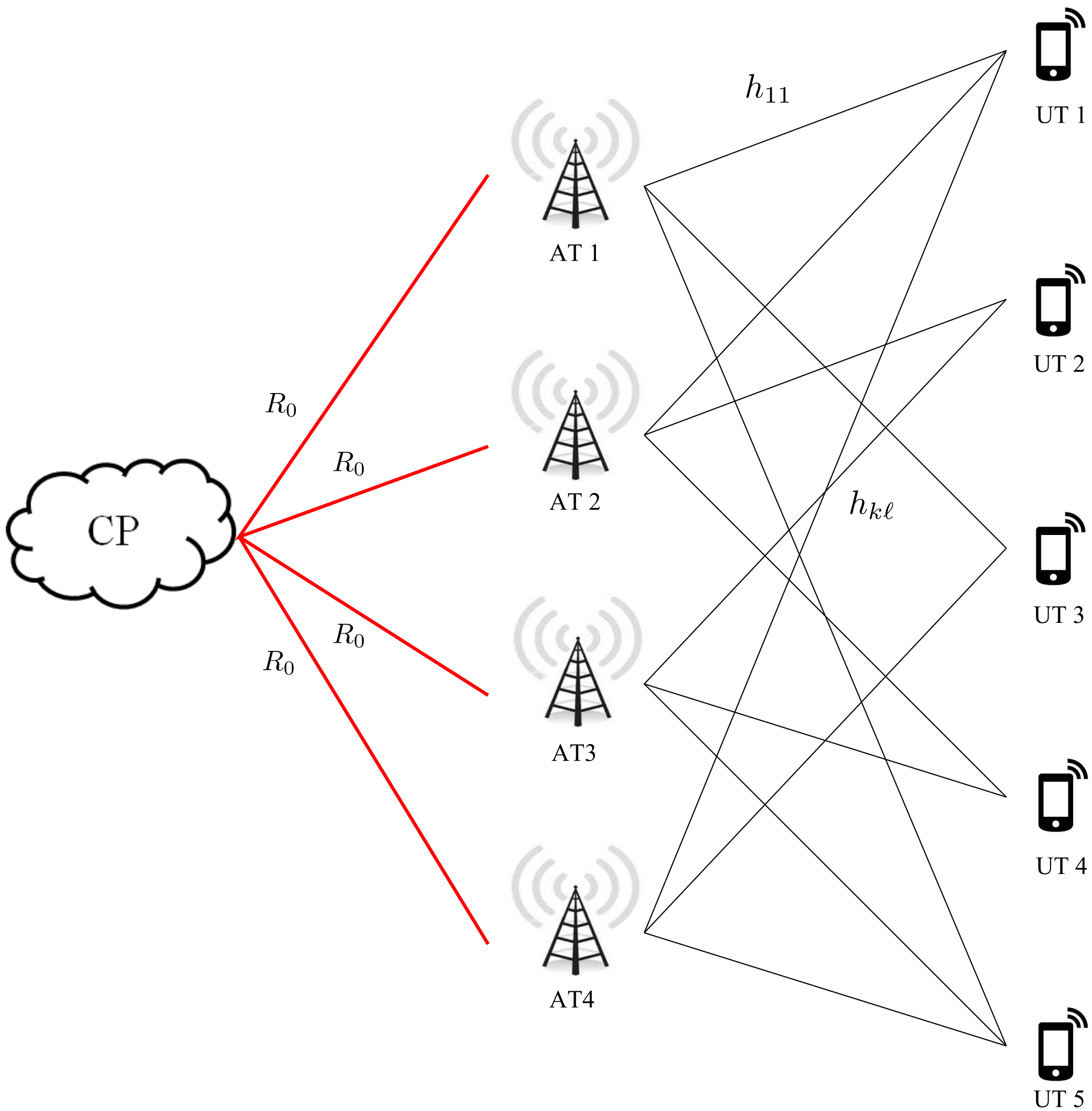
Application to the MAC



Application to the compound MAC

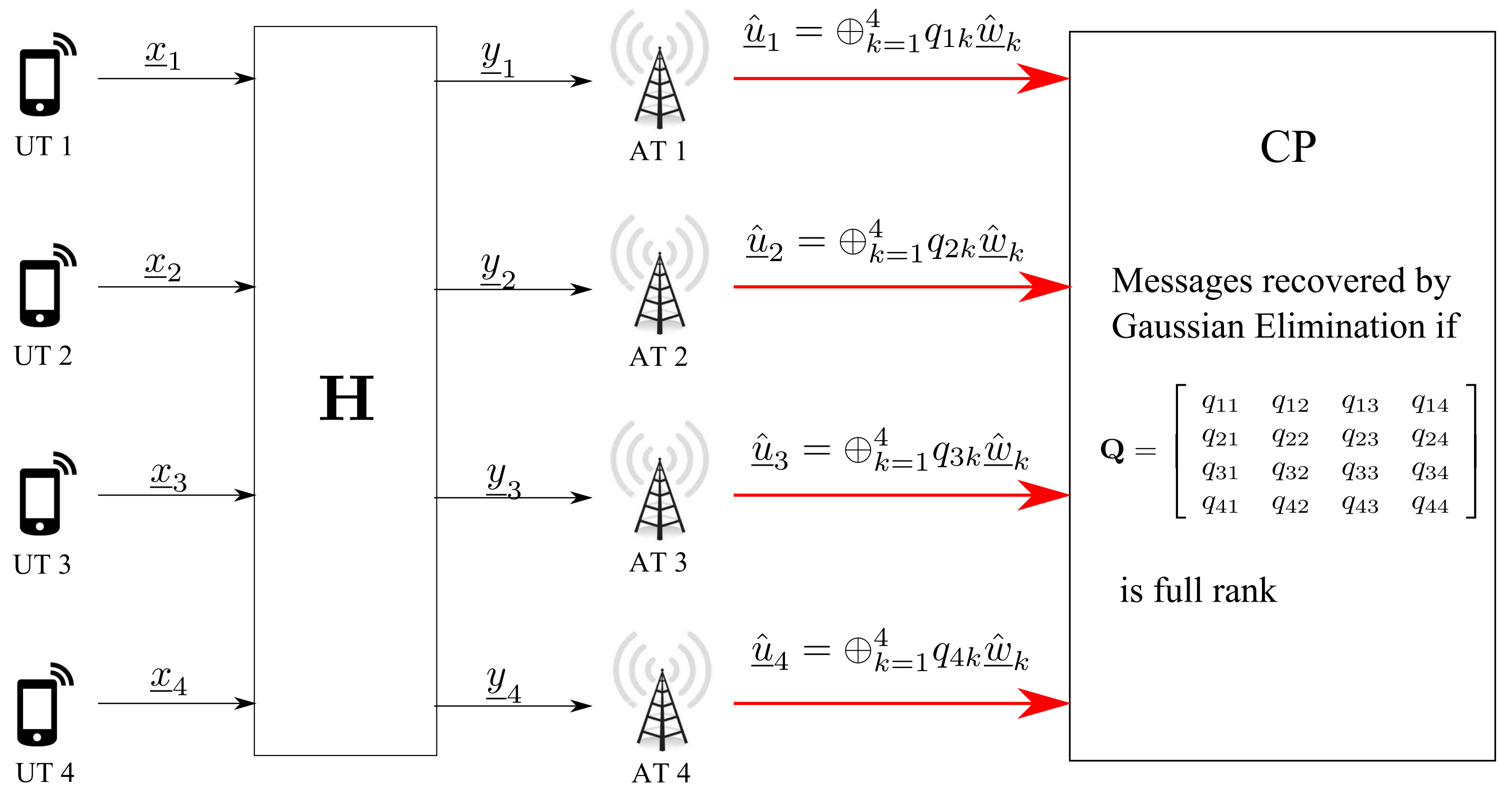


Application of DAS



CommIT
Communications and Information Theory Chair

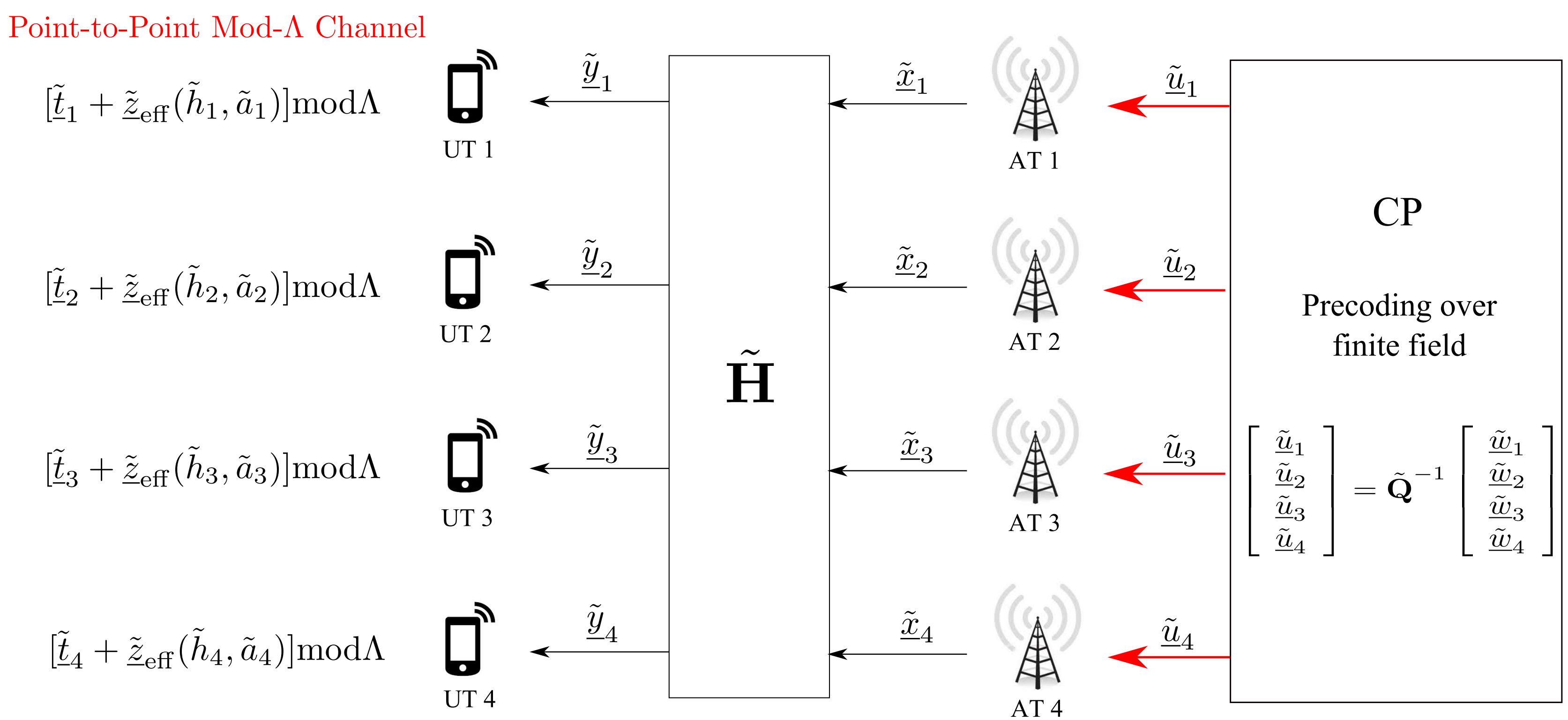
DAS Uplink (MAC-Relay channel)



DAS Downlink (BC-Relay channel)

- We propose a new scheme called Reverse Compute and Forward (RCoF).
- Exchange the role of ATs and UTs.
- Have each UT decode a finite-field linear combination of the messages.
- Precode the messages in the finite-field domain, without power penalty at the transmitter (we do ZF in the finite-field domain).
- Natural competitors: compressed DPC, compressed ZF (precoding over $\mathbb{C}$ and then quantization).

Reverse Compute and Forward



RCoF over CoF: no need for a common lattice code

- The CP forms the messages $\underline{\tilde{\mathbf{w}}}_\ell \in \mathbb{Z}_p^{k_F}$ by appending $k_F - k_{F,\ell}$ zeros to each ℓ-th information message of $k_{F,\ell}$ symbols, so that all messages have the same length.

- The CP produces the precoded messages

$$\begin{bmatrix} \underline{\tilde{\boldsymbol{\mu}}}_1 \\ \vdots \\ \underline{\tilde{\boldsymbol{\mu}}}_L \end{bmatrix} = \tilde{\mathbf{Q}}^{-1} \begin{bmatrix} \underline{\tilde{\mathbf{w}}}_1 \\ \vdots \\ \underline{\tilde{\mathbf{w}}}_L \end{bmatrix}.$$

- The CP forwards the precoded message $\underline{\tilde{\boldsymbol{\mu}}}_\ell$ to AT ℓ for all $\ell = 1, \ldots, L$, via the digital backhaul link.

Communications and Information Theory Chair

- AT ℓ locally produces the lattice codeword $\underline{\boldsymbol{\nu}}_\ell = f(\underline{\tilde{\boldsymbol{\mu}}}_\ell) \in \mathcal{L}_F$ (the densest lattice code), and transmits the corresponding channel input $\underline{\tilde{\mathbf{x}}}_\ell$ by dithering and mod Λ_C.

- Because of lattice/field linearity we can write

$$\begin{bmatrix} \underline{\tilde{\boldsymbol{\nu}}}_1 \\ \vdots \\ \underline{\tilde{\boldsymbol{\nu}}}_L \end{bmatrix} = \mathbf{B} \begin{bmatrix} \underline{\tilde{\mathbf{t}}}_1 \\ \vdots \\ \underline{\tilde{\mathbf{t}}}_L \end{bmatrix} \mod \Lambda_C$$

for some integer matrix $\mathbf{B}$.

Numerical experiments

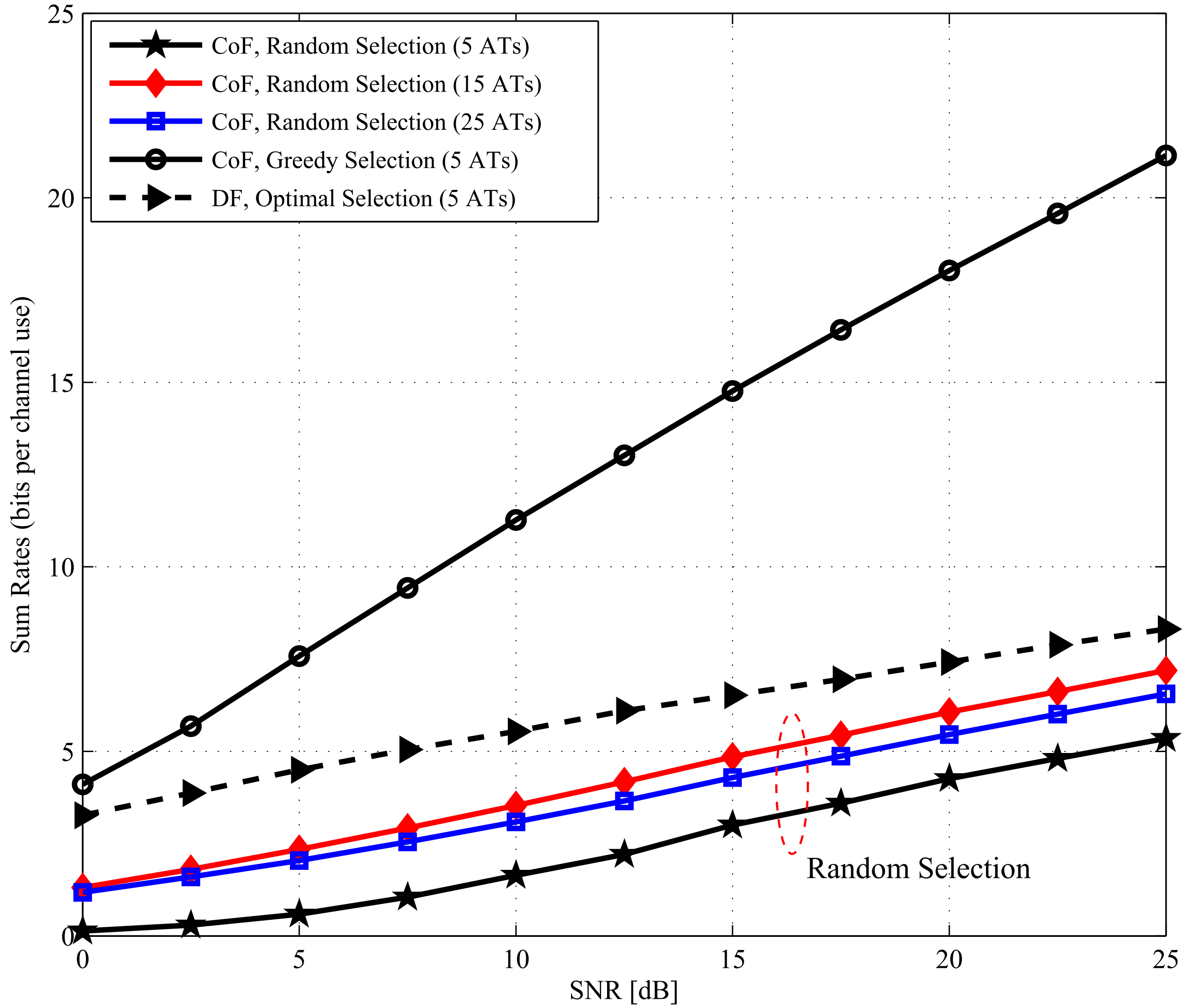
- Bernoulli-Gaussian channel coefficients (Rayleigh fading with blocking probability $\rho = 0.5$).

- $R_0 = 6$ bit/channel use (e.g., for a wireless channel with 20 MHz bandwidth this yields 120 Mb/s).

- Uplink with AT selection: for $L > K$ we select a subset of K ATs in order to maximize the computation rate.

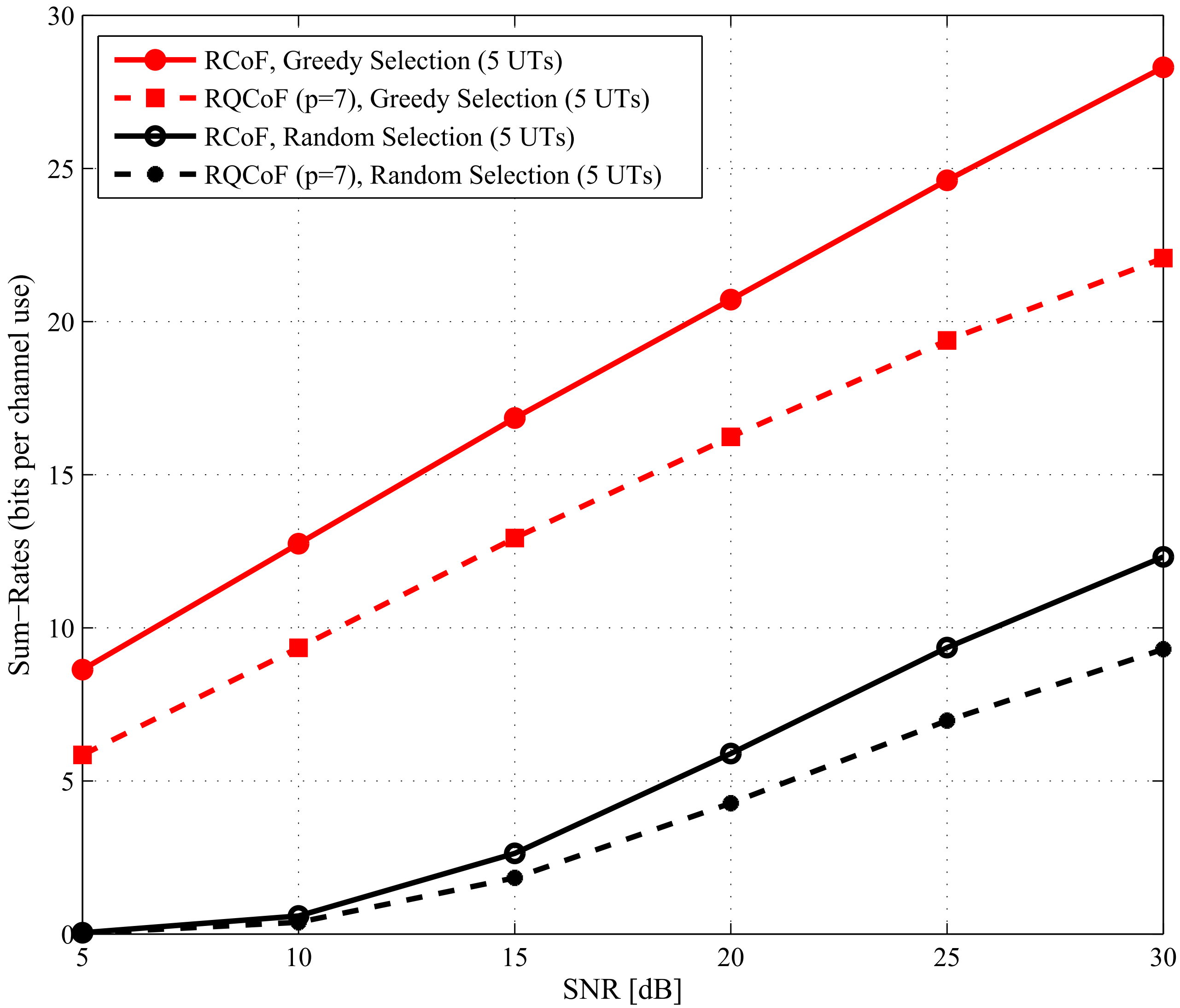
- Downlink with UT selection: for $K > L$ we serve a subset of L UTs in order to maximize the sum rate.

DAS uplink with $K = 5$, $L = 25$



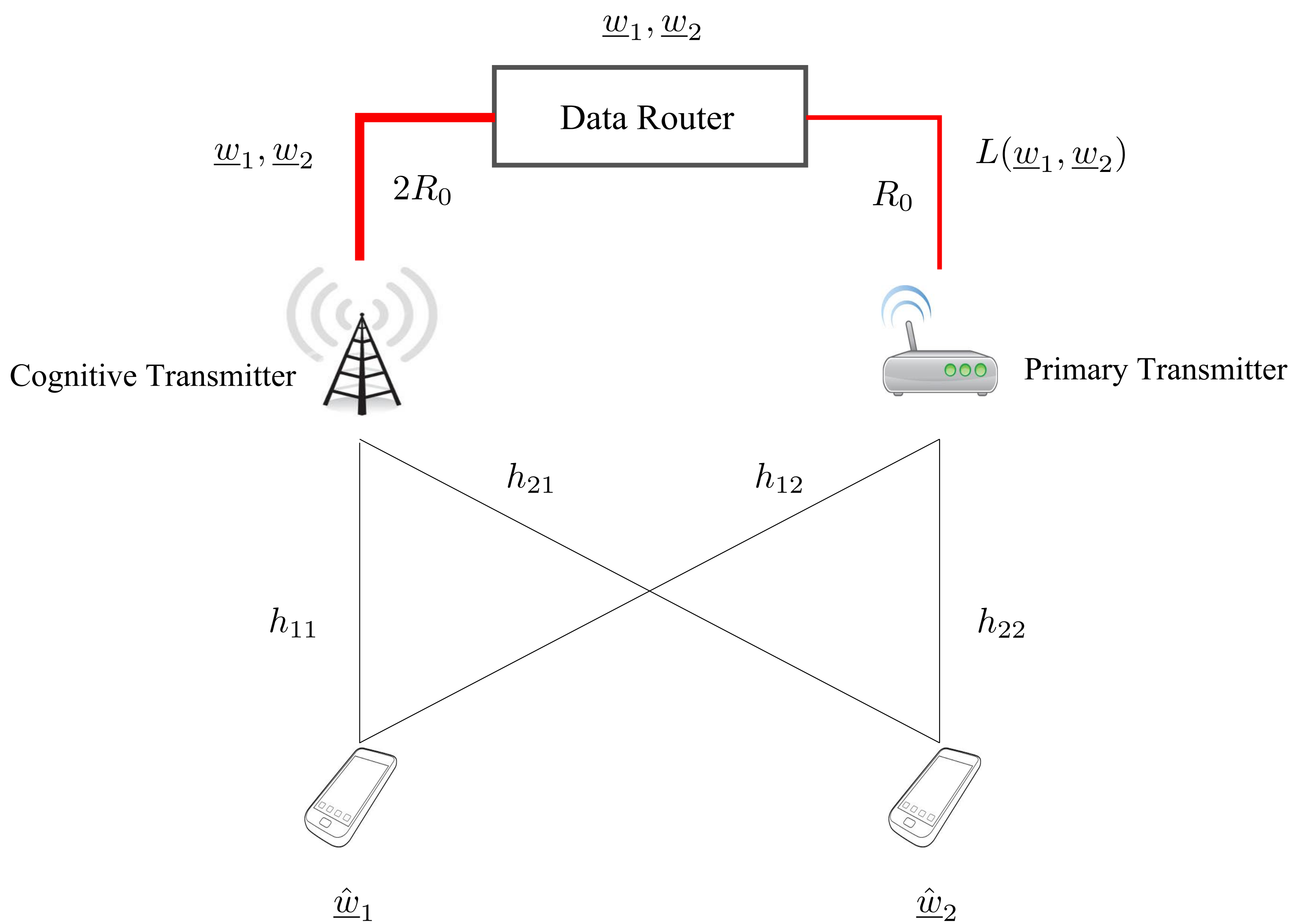
CommIT
Communications and Information Theory Chair

DAS downlink with $K = 25$, $L = 5$



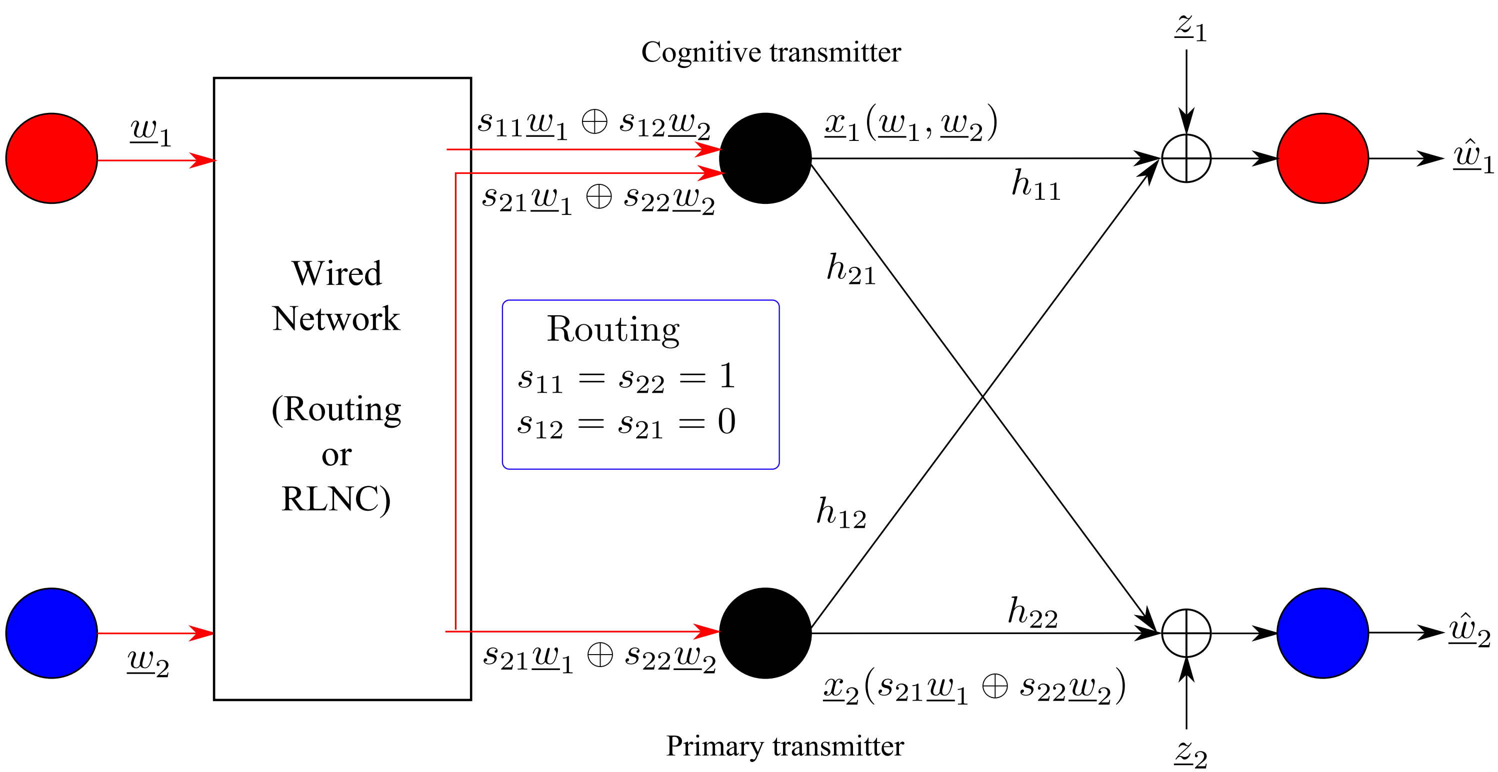
Communications and Information Theory Chair

Network-Coded Cognitive IC



CommIT Communications and Information Theory Chair

Network-Coded Cognitive IC (model)



An achievable rate region

- Transmitter 2 produces the lattice codeword $\underline{\mathbf{v}}_2 = f(\underline{\mathbf{w}}_1 \oplus \underline{\mathbf{w}}_2)$ and produces the channel input

$$\underline{\mathbf{x}}_2' = \beta \underline{\mathbf{x}}_2 = \beta \left([\underline{\mathbf{v}}_2 + \underline{\mathbf{d}}_2] \mod \Lambda \right)$$

- Transmitter 1 produces the precoded message $m\underline{\mathbf{w}}_1$ where $m = (q_1)^{-1}(-q_2)$, where $\mathbf{b} = [b_1, b_2] \in \mathbb{Z}[j]^2$ denotes the integer vector used at receiver 2 for the CoF mapping, and $q_k = [b_k]_q$.

- Transmitter 1 uses DPC for the known interference signal $h_{12}\underline{\mathbf{x}}_2'$ and forms:

$$\underline{\mathbf{x}}_1 = [\underline{\mathbf{v}}_1 - \alpha_1 (h_{12}/h_{11}) \underline{\mathbf{x}}_2' + \underline{\mathbf{d}}_1] \mod \Lambda,$$

where $\underline{\mathbf{v}}_1 = f(m\underline{\mathbf{w}}_1)$.

- Because of linearity, the precoding and the encoding over the finite-field commute. Therefore, we can write

$$\begin{aligned} \underline{\mathbf{v}}_1 &= g(m)\underline{\mathbf{t}}_1 \mod \Lambda \\ \underline{\mathbf{v}}_2 &= \underline{\mathbf{t}}_1 + \underline{\mathbf{t}}_2 \mod \Lambda \end{aligned}$$

where $\underline{\mathbf{t}}_1 = f(\underline{\mathbf{w}}_1)$ and $\underline{\mathbf{t}}_2 = f(\underline{\mathbf{w}}_2)$.

- From standard DPC, Receiver 1 is successful if

$$R_1 \leq \log(1 + |h_{11}|^2 \text{SNR}).$$

- Letting $\tilde{\mathbf{h}}(\beta) = [h_{21}, \beta\tilde{h}_{22}]$ with $\tilde{h}_{22} = h_{22} - \alpha_{1,\text{MMSE}} h_{12}h_{21}/h_{11}$, Receiver 2 applies CoF with integer coefficients $\mathbf{b}$ and scaling factor $\alpha_2 = b_1/h_{21}$, yielding

$$\begin{aligned}
\hat{\underline{\mathbf{y}}}_2 &= [\alpha_2 \underline{\mathbf{y}}_2 - b_1 \underline{\mathbf{d}}_1 - b_2 \underline{\mathbf{d}}_2] \mod \Lambda \\
&= \left[\mathbf{b}^{\mathsf{T}} \left[\begin{array}{c} \underline{\mathbf{v}}_1 \\ \underline{\mathbf{v}}_2 \end{array} \right] + (b_1 \beta \tilde{h}_{22}/h_{21} - b_2)\underline{\mathbf{u}}_2 + (b_1/h_{21})\underline{\mathbf{z}}_2 \right] \mod \Lambda \\
&= \left[\left(\mathbf{b}^{\mathsf{T}} \left[\begin{array}{cc} g(m) & 0 \\ 1 & 1 \end{array} \right] \mod p\mathbb{Z}[j] \right) \left[\begin{array}{c} \underline{\mathbf{t}}_1 \\ \underline{\mathbf{t}}_2 \end{array} \right] + \underline{\mathbf{z}}_{\text{eff}}(\tilde{\mathbf{h}}(\beta), \mathbf{b}) \right] \mod \Lambda \\
&\overset{(a)}{=} [([b_2] \mod p\mathbb{Z}[j])\underline{\mathbf{t}}_2 + \underline{\mathbf{z}}_{\text{eff}}(\tilde{\mathbf{h}}(\beta), \mathbf{b})] \mod \Lambda
\end{aligned}$$

where $\underline{\mathbf{u}}_2$ is uniformly distributed on $\mathcal{V}_\Lambda$ and is independent of $\underline{\mathbf{v}}_1$, $\underline{\mathbf{v}}_2$, and $\underline{\mathbf{z}}_2$ by the independence and uniformity of dithering and by the Crypto Lemma, $\underline{\boldsymbol{\lambda}} = Q_\Lambda(\underline{\mathbf{v}}_1 - \alpha_{1,\text{MMSE}}\beta(h_{12}/h_{11})\underline{\mathbf{x}}_2 + \underline{\mathbf{d}}_1)$, where (a) follows from the fact that, by construction, $b_1 g(m) + b_2 \mod p\mathbb{Z}[j] = 0$.

- The effective noise is given by

$$\underline{\mathbf{z}}_{\text{eff}}(\tilde{\mathbf{h}}(\beta), \mathbf{b}) = (b_1 \beta \tilde{h}_{22}/h_{21} - b_2)\underline{\mathbf{u}}_2 + (b_1/h_{21})\underline{\mathbf{z}}_2$$

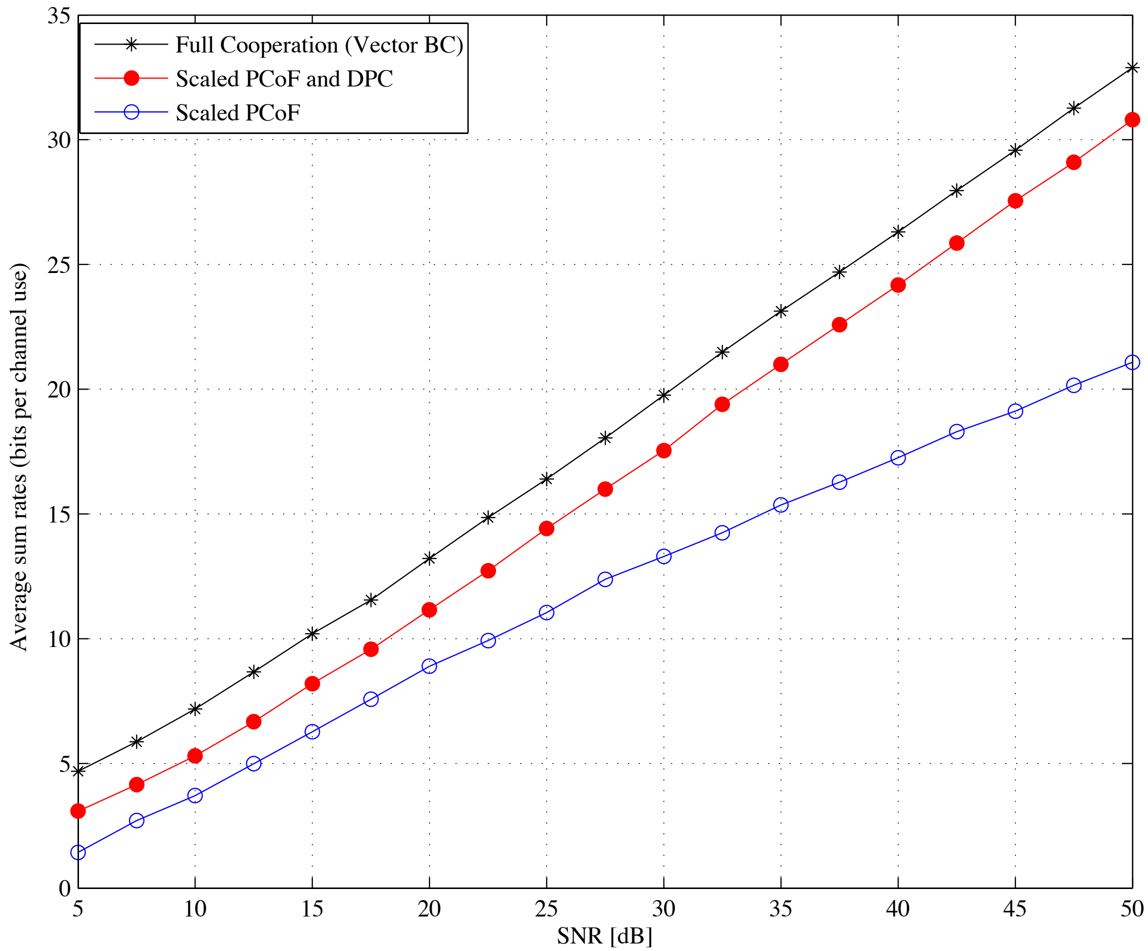
- Receiver 2 decodes $\underline{\mathbf{t}}_2$ by applying lattice decoding to $\hat{\underline{\mathbf{y}}}_2$. This yields the achievable rate

$$R_2 \leq R_{\text{comp}}(\text{SNR}, \sigma^2_{\text{eff}}(\beta))$$

for any $\mathbf{b} \in \mathbb{Z}^2[j]$ with $b_1, b_2 \neq 0 \mod p\mathbb{Z}[j]$ and any $\beta \in \mathbb{R}_+$ satisfying the input power constraint, where

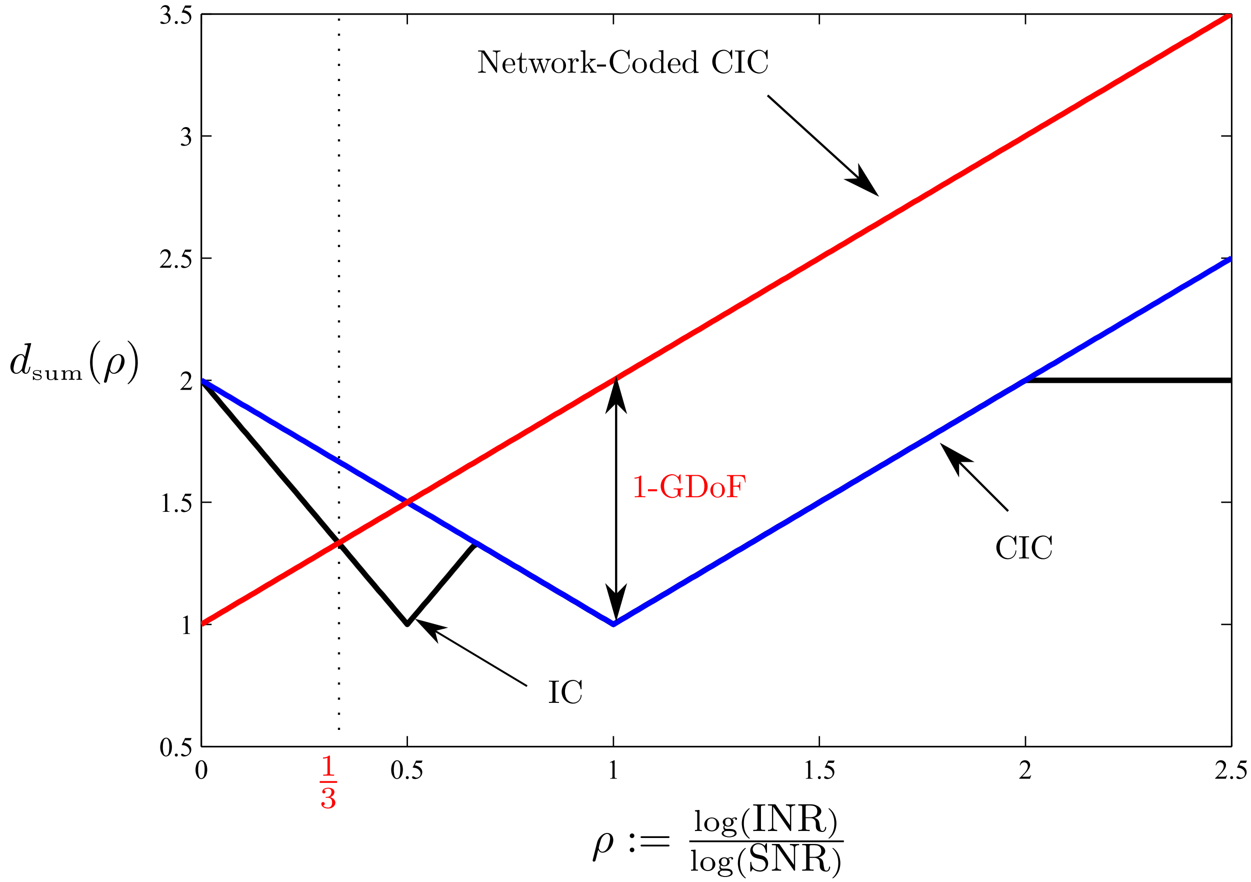
$$\sigma^2_{\text{eff}}(\beta) = \left| b_1 \frac{\beta \tilde{h}_{22}}{h_{21}} - b_2 \right|^2 \text{SNR} + \left| \frac{b_1}{h_{21}} \right|^2.$$

Example: independent Rayleigh fading



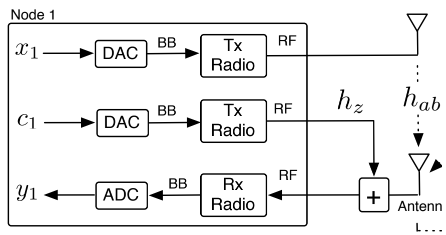
Communications and Information Theory Chair

Symmetric GDoFs

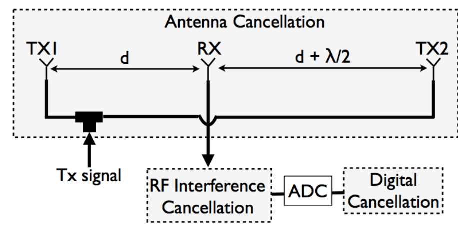


- Dinesh Bharadia, Emily Mcmilin and Sachin Katti, ”Full Duplex Radios” ACM SIGCOMM 2013

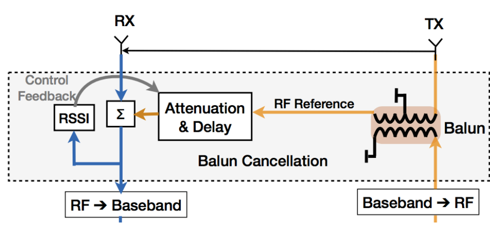
- M. Duarte and A. Sabharwal, ”Full-Duplex Wireless Communications Using Off-The-Shelf Radios: Feasibility and First Results” Asilomar 2010



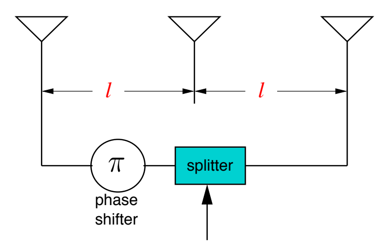
Rice (Duarte et. al, 2010)
Rice, AT&T (Duarte et. al, 2012)



Stanford I (Choi et. al, 2010)



Stanford II (Jain et. al, 2011)



Princeton, NEC (Aryafar et. al, 2012)

Why not distributed?

- Many of the current full-duplex schemes “hide” two or even 3 RF chains in the same box.
- Distance, phase cancellation, or echo-cancellation in the RF domain are used to avoid saturation of the Rx front-end while Tx is active.
- We can obtain the same “full duplex” effect by using separated half-duplex relays.
- T. J. Oechtering and A. Sezgin, ”A new cooperative transmission scheme using the space-time delay code,” ITG Workshop on Smart Antenna 2004.
- B. Rankov and A. Wittneben, ”Spectral Efficient Signaling for Half-Duplex Relay Channels,” Asilomar 2005.
- S. S. C. Rezaei, S. O. Gharan, and A. K. Khandani, ”Cooperative Strategies for the Half-Duplex Gaussian Parallel Relay Channel: Simultaneous Relaying versus Successive Relaying,” Allerton 2008.

Virtual full-duplex relaying

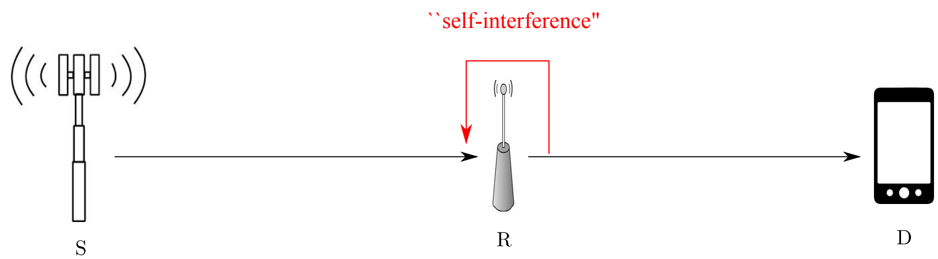


Fig. 1. Two-hop source-relay-destination network with full-duplex relay.

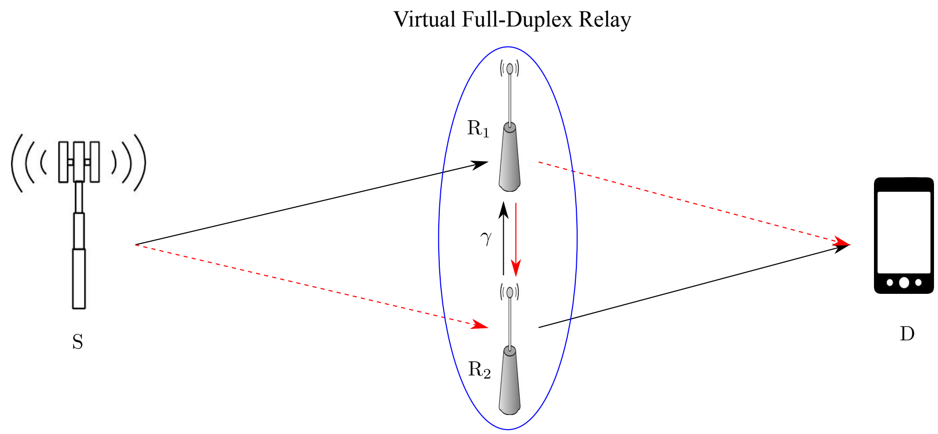


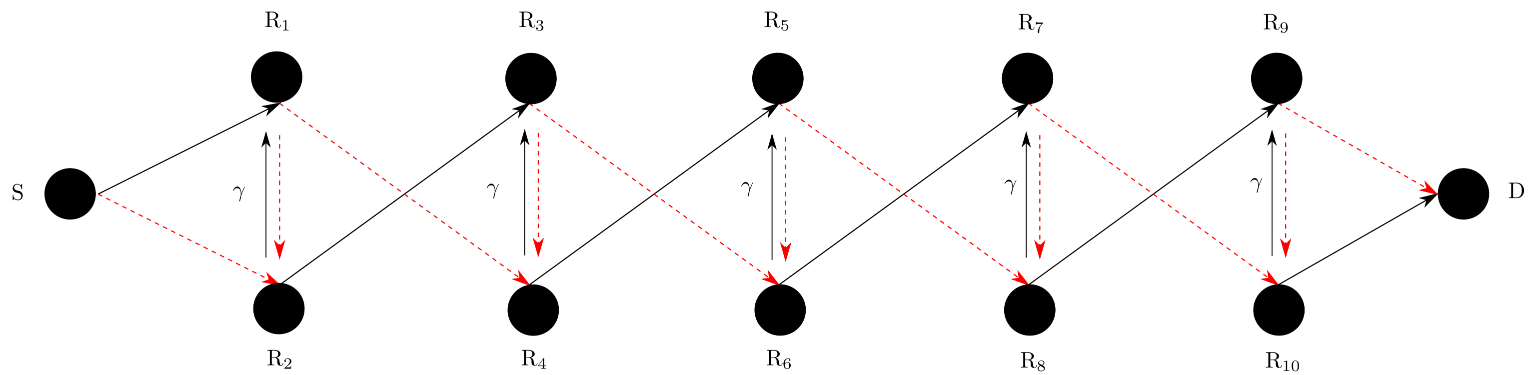
Fig. 2. Virtual full-duplex relay implemented by two half-duplex relays where γ denotes the inter-relay interference.

CoF with forward substitution

	time slot 1	time slot 2	time slot 3	time slot 4
X_S	$\underline{\mathbf{x}}_S(\underline{\mathbf{w}}_1)$	$\underline{\mathbf{x}}_S(\underline{\mathbf{w}}_2)$	$\underline{\mathbf{x}}_S(\underline{\mathbf{w}}_3)$	$\underline{\mathbf{x}}_S(\underline{\mathbf{w}}_4)$
Y_{R_1}		$\underline{\mathbf{u}}_2 = q_1\underline{\mathbf{w}}_2 \oplus q_2\underline{\mathbf{u}}_1$		$\underline{\mathbf{u}}_4 = q_1\underline{\mathbf{w}}_4 \oplus q_2\underline{\mathbf{u}}_3$
X_{R_1}			$\underline{\mathbf{x}}_R(\underline{\mathbf{u}}_2)$	
Y_{R_2}	$\underline{\mathbf{u}}_1 = \underline{\mathbf{w}}_1$		$\underline{\mathbf{u}}_3 = q_1\underline{\mathbf{w}}_3 \oplus q_2\underline{\mathbf{u}}_2$	
X_{R_2}		$\underline{\mathbf{x}}_R(\underline{\mathbf{u}}_1)$		$\underline{\mathbf{x}}_R(\underline{\mathbf{u}}_3)$
Y_D		$\underline{\hat{\mathbf{w}}}_1 = \underline{\mathbf{u}}_1$	$\underline{\hat{\mathbf{w}}}_2 = q_1^{-1}\underline{\mathbf{u}}_2 \ominus q_1^{-1}q_2\underline{\mathbf{u}}_1$	$\underline{\hat{\mathbf{w}}}_3 = q_1^{-1}\underline{\mathbf{u}}_3 \ominus q_1^{-1}q_2\underline{\mathbf{u}}_2$

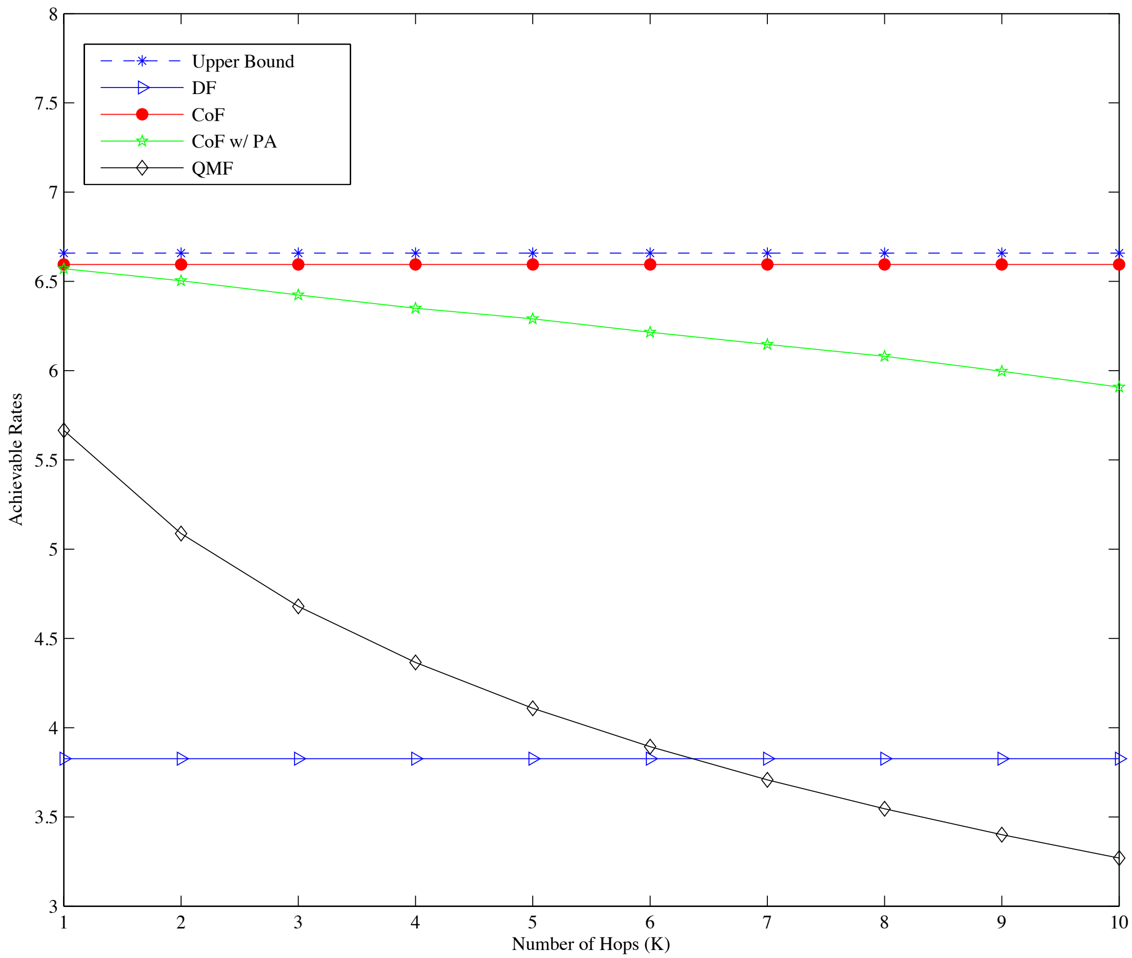
- Each receiving relay decodes an integer combination of the source codeword and of the transmitting relay codeword.
- The integer combinations are forwarded to the destination, which can decode them by forward substitution (no large decoding latency as in backward decoding).

Extension to multi-hop relaying

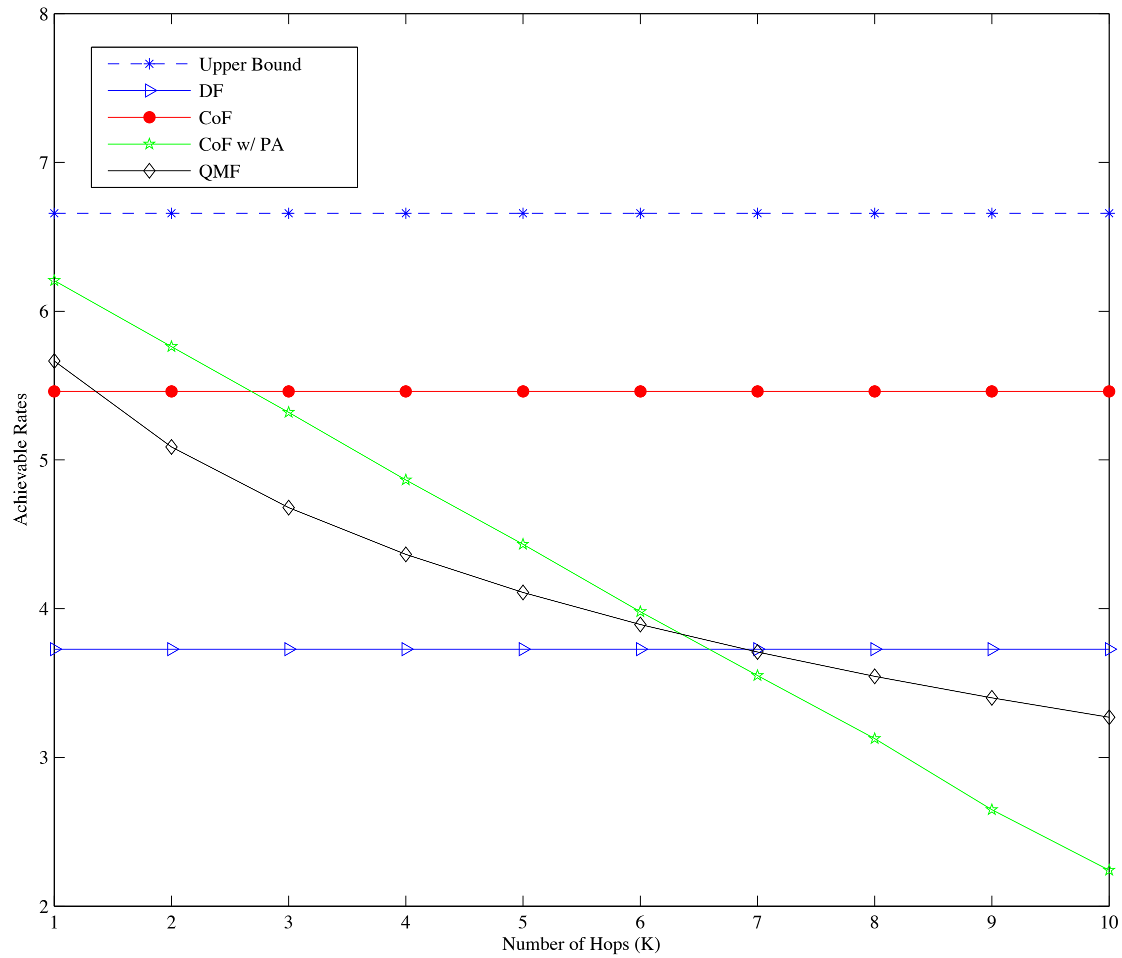


- A possible motivation: wireless backhaul in mm-wave small cell networks.
- Line-of-sight point to point links to connect small cell base stations deployed without the need of putting down cable/fiber.

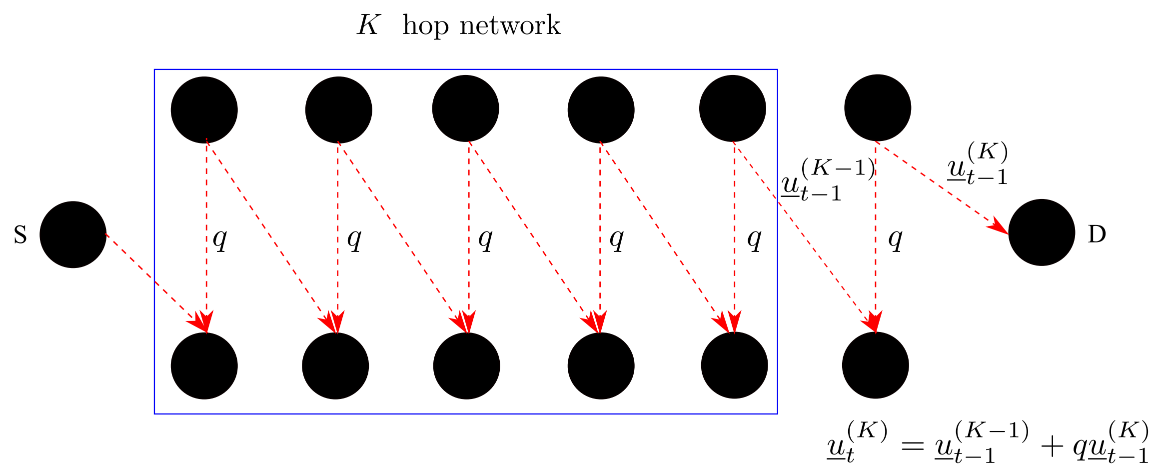
Results for SNR $= 20$ dB and $\gamma^2 \sim$ Unif$(0.9, 1.1)$



Results for SNR $= 20$ dB and $\gamma^2 \sim$ Unif$(0.5, 1.0)$



Remarks: K-hops CoF with forward substitution



- The goal is to obtain at the destination a lower-triangular full rank system of (noiseless) equations over the message finite field.

- Since the destination begins to receive a signal after K time slots, we have

$$\underline{\mathbf{u}}_t^{(K)} = 0 \text{ for } t \leq K.$$

- We also have that $\underline{\mathbf{u}}_{K+1}^{(K)} = \underline{\mathbf{w}}_1$ since the first signal is not interfered.
- In case of $K = 1$, we can easily compute the following relation:

$$\underline{\mathbf{u}}_{t+1}^{(1)} = \sum_{\ell=1}^{t} q^{t-\ell} \underline{\mathbf{w}}_\ell$$

- At time slot $t+1$, the above equation has only one unknown $\underline{\mathbf{w}}_t$ since the destination has been already decoded $\{\underline{\mathbf{w}}_\ell : \ell = 1, \ldots, t-1\}$ during the previous time slots. Thus, it can recover the desired message $\underline{\mathbf{w}}_t$ such as

$$\underline{\mathbf{w}}_t = \underline{\mathbf{u}}_{t+1}^{(1)} - \sum_{\ell=1}^{t-1} q^{t-\ell} \underline{\mathbf{w}}_\ell = \underline{\mathbf{u}}_{t+1}^{(1)} - q \underline{\mathbf{u}}_t^{(1)}.$$

- For the case $K \geq 2$, we can derive the following relation:

$$\underline{\mathbf{u}}_{t+1}^{(K)} - q\underline{\mathbf{u}}_{t}^{(K)} = \underline{\mathbf{u}}_{t}^{(K-1)}$$

- For example, when $K = 3$, we have:

$$\begin{aligned} A = \underline{\mathbf{u}}_{t+3}^{(3)} - q\underline{\mathbf{u}}_{t+2}^{(3)} &= \underline{\mathbf{u}}_{t+2}^{(2)} \\ B = (\underline{\mathbf{u}}_{t+2}^{(3)} - q\underline{\mathbf{u}}_{t+1}^{(3)}) &= \underline{\mathbf{u}}_{t+1}^{(2)}. \end{aligned}$$

such that

$$A - qB = \underline{\mathbf{u}}_{t+3}^{(3)} - 2q\underline{\mathbf{u}}_{t+2}^{(3)} + q^2\underline{\mathbf{u}}_{t+1}^{(3)} = \underline{\mathbf{u}}_{t+1}^{(1)} = \sum_{\ell=1}^{t} q^{t-\ell}\underline{\mathbf{w}}_{\ell}.$$

- At time slot $t+3$, the destination can decode $\underline{\mathbf{w}}_t$ using previously decoded messages $\{\underline{\mathbf{w}}_\ell : \ell = 1, \ldots, t-1\}$ and observations $\{\underline{\mathbf{u}}_\ell : \ell = 1, \ldots, t+3\}$ so that

$$\underline{\mathbf{w}}_t = \underline{\mathbf{u}}_{t+3}^{(3)} - 2q\underline{\mathbf{u}}_{t+2}^{(3)} + q^2\underline{\mathbf{u}}_{t+1}^{(3)} - \sum_{\ell=1}^{t-1} q^{t-\ell}\underline{\mathbf{w}}_\ell.$$

from which (after some algebra) we obtain the sliding-window decoding:

$$\underline{\mathbf{w}}_t = \underline{\mathbf{u}}_{t+3}^{(3)} - 3q\underline{\mathbf{u}}_{t+2}^{(3)} + 3q^2\underline{\mathbf{u}}_{t+1}^{(3)} - q^3\underline{\mathbf{u}}_t^{(3)}.$$

- By induction, we can prove:

Lemma:

For the $(K+1)$-hop network with CoF, the following relation holds:

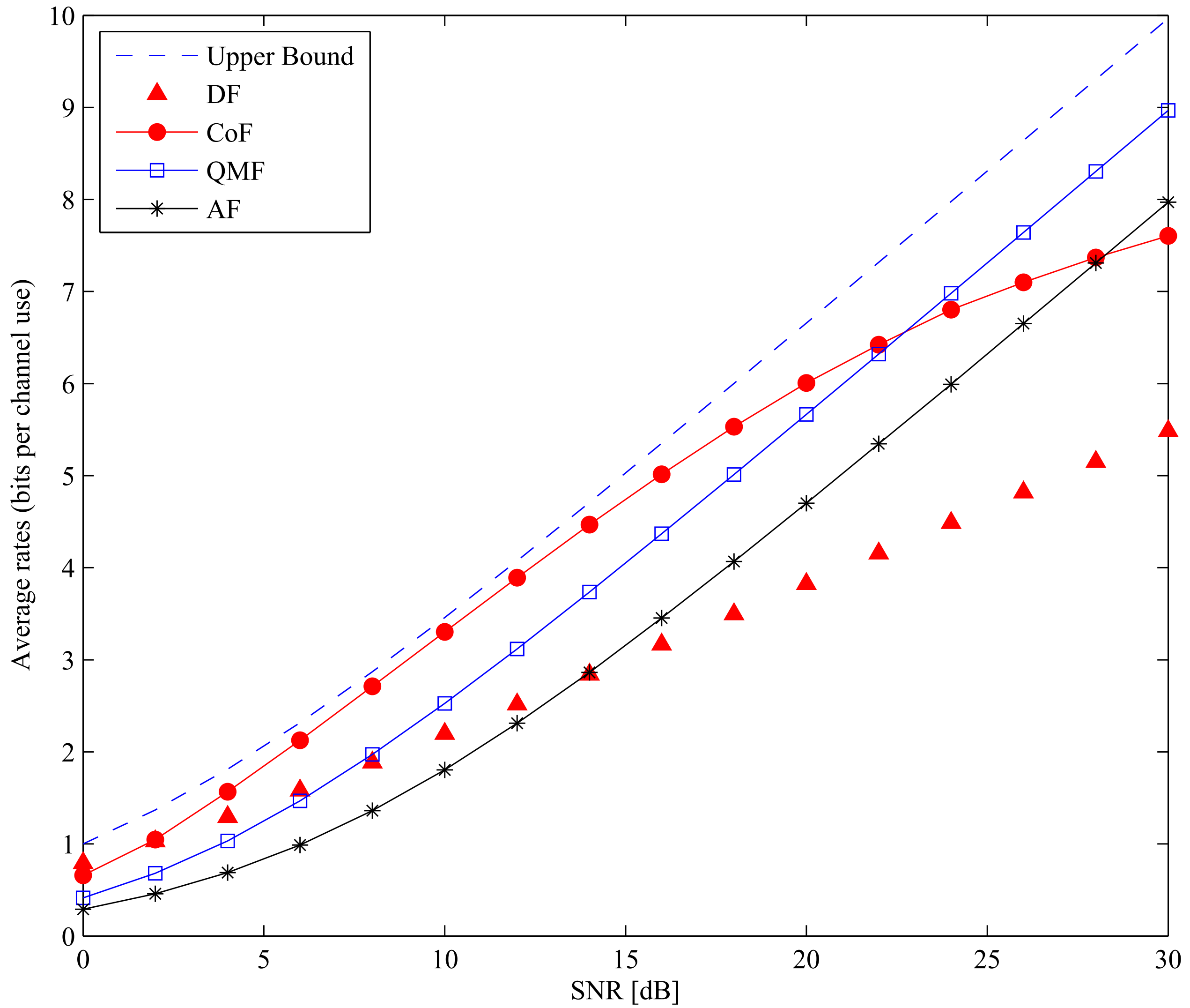
$$\sum_{\ell=1}^{K}(-q)^{\ell-1}\begin{pmatrix} K-1 \\ \ell-1 \end{pmatrix}\underline{\mathbf{u}}_{t-\ell+K+1}^{(K)} = \sum_{\ell=1}^{t} q^{t-\ell}\underline{\mathbf{w}}_{\ell}.$$

Hence, the destination can decode the desired message $\underline{\mathbf{w}}_t$ at time slot $t+K$ by ways of

$$\underline{\mathbf{w}}_t = \sum_{\ell=1}^{K+1}(-q)^{\ell-1}\binom{K}{\ell-1}\underline{\mathbf{u}}_{t-\ell+K+1}^{(K)}.$$

Communications and Information Theory Chair

Results


Upper Bound
DF
CoF
QMF
AF
Average rates (bits per channel use)
SNR [dB]
0
1
2
3
4
5
6
7
8
9
10
0
5
10
15
20
25
30

Conclusions

- CoF, RCoF, QCoF, QRCoF, PCoF with channel integer alignment (CIA) ... too many acronyms!.
- General idea: quench the noise at the intermediate nodes, generate a deterministic linear finite field channel over which we can precode without power penalty.
- Non-integer penalty: sometimes it can be eliminated by "channel integer alignment".
- Minimum common rate (same lattice code) can be alleviated (e.g., by RCoF, unequal power, unequal rate ...).
- For each of these schemes, a low-complexity implementation based on scalar quantization and q-ary linear coding is possible (at the cost of the shaping gain for large q).

Thank You