# Time-Varying Systems and Computations

## Unit 9.1

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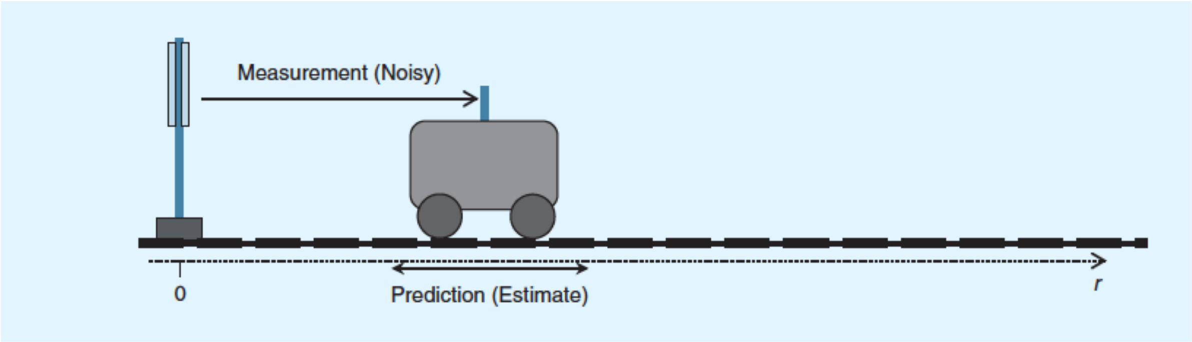
# One-dimensional system (example)




- State vector

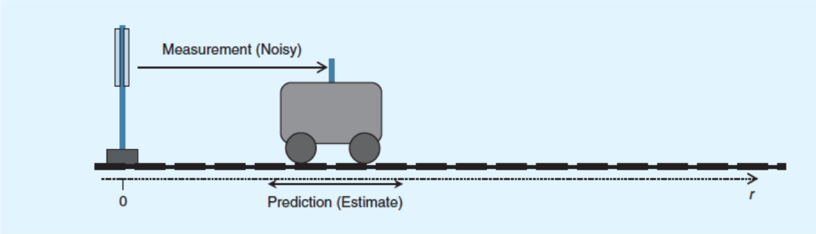
$$x_k = \begin{bmatrix} s_k \\ v_k \end{bmatrix} \begin{matrix} \text{position} \\ \text{velocity} \end{matrix}$$

- Noisy measurement of position
- Physical model for motion of the cart

# One-dimensional system

- Physical Model

$$f_k \quad \text{Force}$$
$$u_k = f_k/m \quad \text{Acceleration}$$
$$m \quad \text{Mass}$$



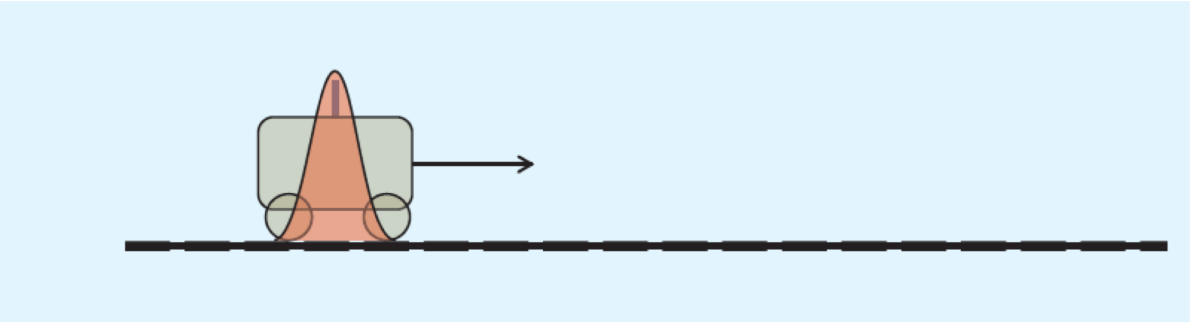

- State equations (model of motion) for prediction

$$\begin{bmatrix} s_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s_k \\ v_k \end{bmatrix} + \begin{bmatrix} \frac{(\Delta t)^2}{2} \\ \Delta t \end{bmatrix} \frac{f_k}{m}$$

# Initial knowledge of the system

Time *k=0*

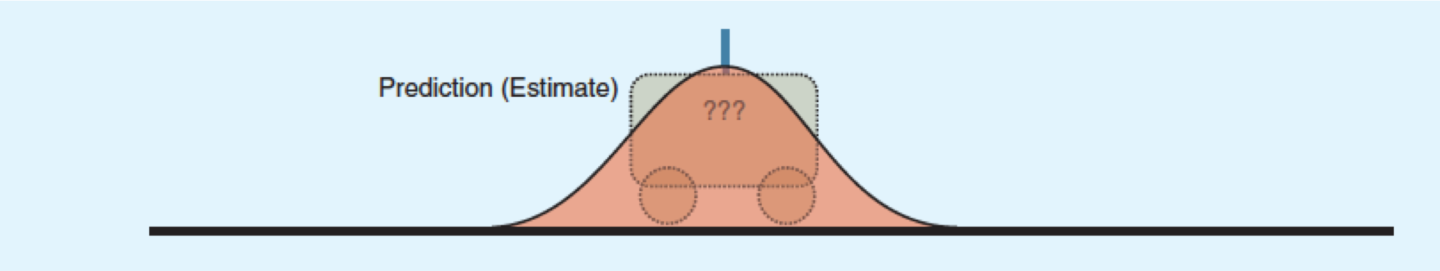
- Initial position with uncertainty
- Initial velocity (indicated by arrow)
- Model for motion of train is known



# Prediction of position and speed

Time *k=1*

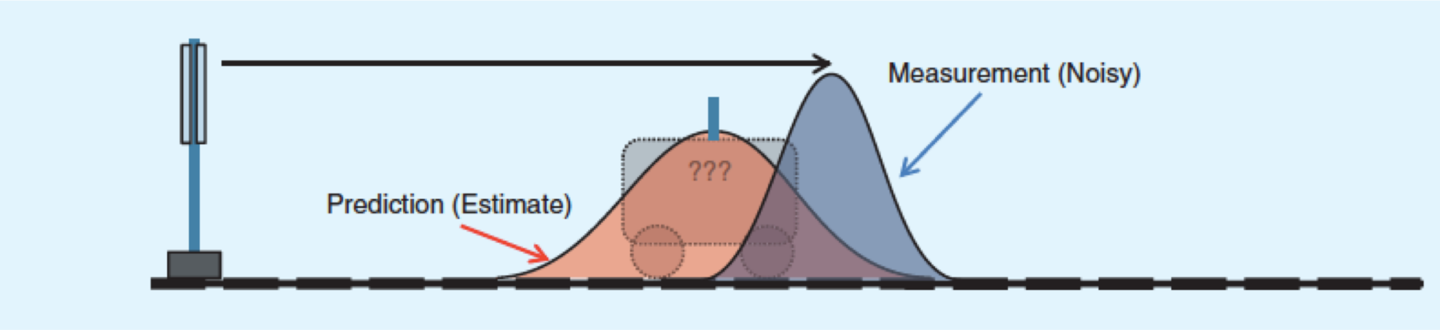
- Prediction of location with uncertainty
- Uncertainty modeled by Gaussian pdf
- Uncertainty if cart was accelerated




$$y_1(r, \mu_1, \sigma_1) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(r-\mu_1)^2}{2\sigma_1^2}}$$

# Measurement of position

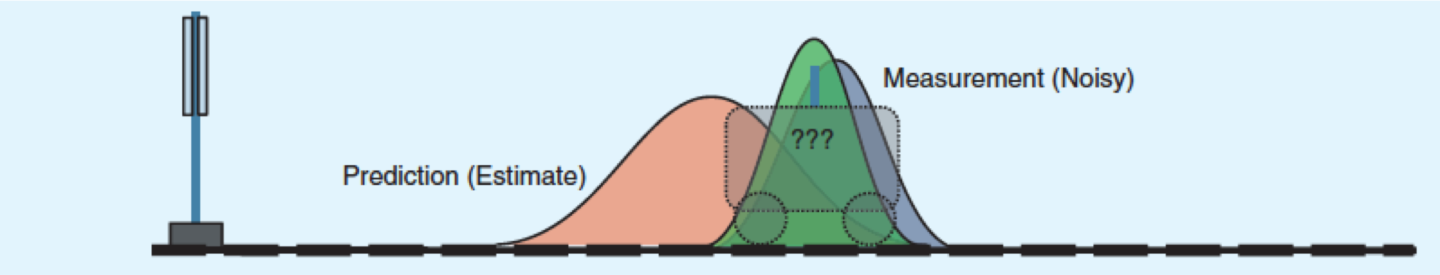
- Noisy measurement of position
- Measurement noise modeled with Gaussian pdf




$$y_2(r, \mu_2, \sigma_2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(r-\mu_2)^2}{2\sigma_2^2}}$$

# Measurement and Prediction

- Combination of measurement and prediction
- Product of two Gaussian pdfs → Gaussian pdf




$$y_{fused}(r, \mu_1, \sigma_1, \mu_2, \sigma_2) = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{(r-\mu_1)^2}{2\sigma_1^2}} \cdot \frac{1}{\sqrt{2\pi\sigma_2^2}} e^{-\frac{(r-\mu_2)^2}{2\sigma_2^2}}$$

$$= \frac{1}{2\pi\sqrt{\sigma_1^2\sigma_2^2}} e^{-\left(\frac{(r-\mu_1)^2}{2\sigma_1^2} + \frac{(r-\mu_2)^2}{2\sigma_2^2}\right)}$$

# Parameters of Gaussian pdf

Most likely position

$$y_{fused}(r, \mu_{fused}, \sigma_{fused}) = \frac{1}{\sqrt{2\pi\sigma_{fused}^2}} e^{-\frac{(r-\mu_{fused})^2}{2\sigma_{fused}^2}}$$

Uncertainty

$$\mu_{fused} = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = \mu_1 + \frac{\sigma_1^2(\mu_2 - \mu_1)}{\sigma_1^2 + \sigma_2^2}$$

$$\sigma_{fused}^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \sigma_1^2 - \frac{\sigma_1^4}{\sigma_1^2 + \sigma_2^2}$$

Update equations