

Time-Varying Systems and Computations

Unit 7.4

Klaus Diepold

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Allpass – Minimum Phase Decomposition

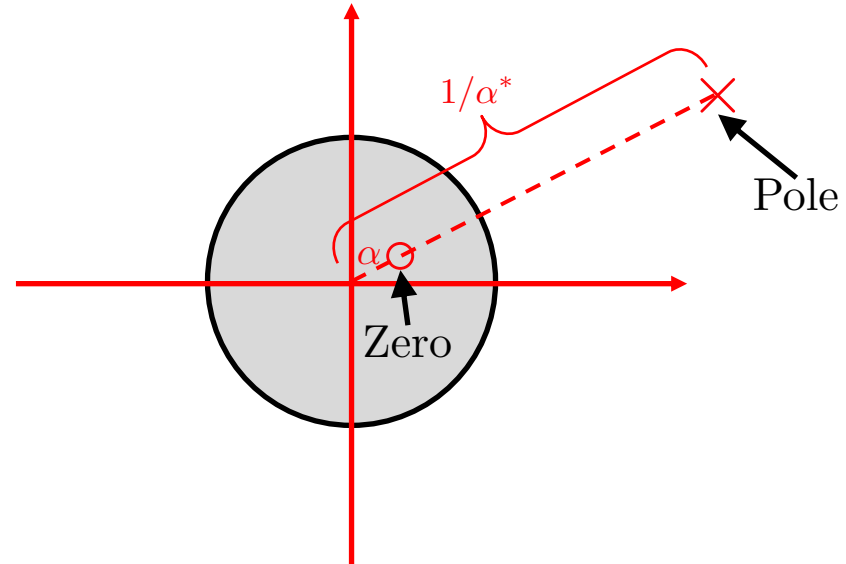
- Allpass Function – Inner Function

$$T_i(z) = \frac{z - \alpha}{1 - \alpha^* z} \quad |\alpha| < 1$$

- Allpass Property

$$|T_{AP}(z)|_{z=e^{j\omega t}} = 1$$

- Not causally invertible
- Pure phase function (delay)

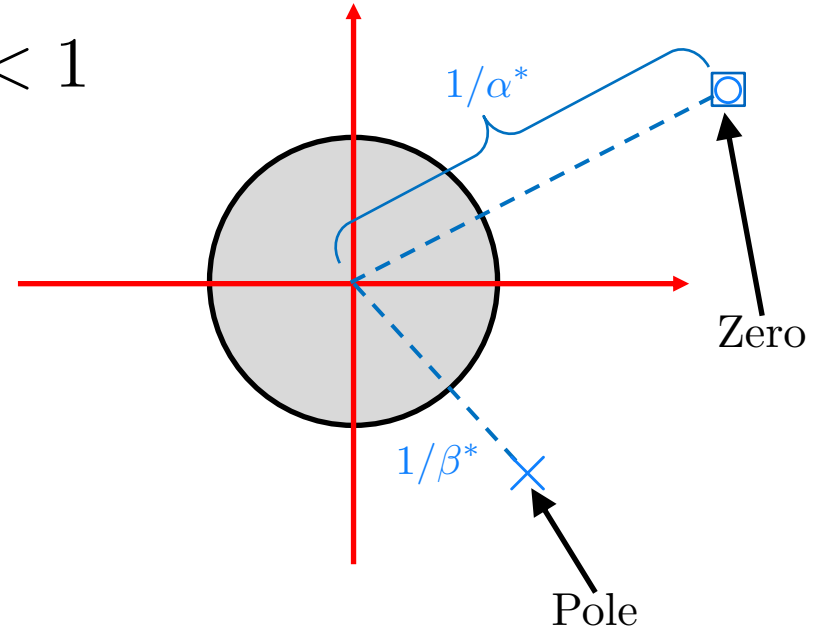


Allpass – Minimum Phase Decomposition

- Minimum Phase Function – Outer Function

$$T_o(z) = \frac{1 - \alpha^* z}{1 - \beta^* z} \quad |\alpha, \beta| < 1$$

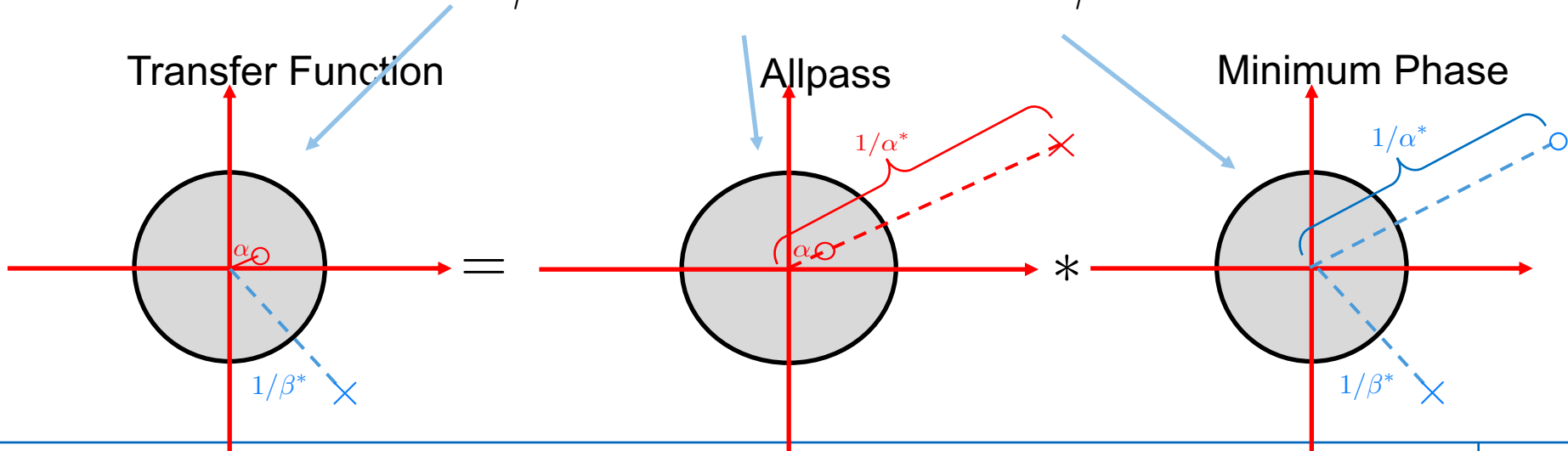
- Causally invertible
- Poles \rightarrow Zeros, Zeros \rightarrow Poles
Outside unit disk



Allpass – Minimum Phase Decomposition

- Transfer Function $T(z) = T_i(z) \cdot T_o(z)$

$$T(z) = \frac{z - \alpha}{1 - \beta^* z} = \frac{z - \alpha}{1 - \alpha^* z} \cdot \frac{1 - \alpha^* z}{1 - \beta^* z}$$



Allpass – Minimum Phase Decomposition



- Very common operation for LTI systems
- Used for approximate inversion of transfer functions
- E.g. in audio signal processing (3D audio)
- Direction of arrival estimation for telepresence applications