Time-Varying Systems and Computations

Unit 7.2

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WS 2024

Realization of an Inner Matrix

- Given an Inner Matrix Q with $Q^T Q = 1$ …

- … find orthogonal (lossless) realization with $\Sigma_k^T \Sigma_k = 1$ such that

$$Q = D + C(1 - AZ)^{-1} ZB \qquad \Sigma_k = \left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right]$$

- Assume we have ANY non-orthogonal realization for Q include state-space equivalence transformation

$$\widehat{\Sigma}_k = \left[\begin{array}{c|c} S_{k+1} A_k S_k^{-1} & S_k B_k \\ \hline C_k S_k^{-1} & D_k \end{array}\right]$$

Realization of an Inner Matrix

- Determine State-Space Equivalence Transformation to make $\widehat{\Sigma}_k$ an orthogonal realization $\widehat{\Sigma}_k^T \widehat{\Sigma}_k = 1$
- Orthogonality conditions

$$S_k^{-T} A_k^T S_{k+1}^T S_{k+1} A_k S_k^{-1} + S_k^{-T} C_k^T C_k S_k^{-1} = 1$$

$$B_k^T S_{k+1}^T S_{k+1} B_k + D_k^T D_k = 1$$

$$B_k^T S_{k+1}^T S_{k+1} A_k S_k + D_k^T C_k S_k = 0$$

- use substitution

$$M_k = S_k^T S_k$$

Realization of an Inner Matrix

- Solve Lyapunov equations for $M_k = M_k^T > 0$

$$A_k^T M_{k+1} A_k + C_k^T C_k = M_k$$

$$B_k^T M_{k+1} B_k + D_k^T D_k = 1$$

$$B_k^T M_{k+1} A_k + D_k^T C_k = 0$$

- Cholesky Factorization of M_k provides for necessary state trafo …

$$M_k = S_k^T S_k$$

- … to make realization $\widehat{\Sigma}_k$ orthogonal

Realization of an Inner Matrix

- Inner Matrix Q → we can find an orthogonal realization $\widehat{\Sigma}_k$
- Finding an orthogonal realization can be done via solving Lyapunov equations and using Cholesky factorization
- Better: Finding an orthogonal realization via a square-root algorithm (see book for details)