Time-Varying Systems and Computations

Unit 7.1

Klaus Diepold

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Inner Matrix

- Causal & Isometric (lower triangular & orthogonal)

$$T^T T = 1 \quad \& \quad T \text{ lower triangular}$$

$$\Rightarrow T = \begin{bmatrix} \pm 1 & & & \\ & \pm 1 & & \\ & & \ddots & \\ & & & \pm 1 \end{bmatrix} \; ?$$

$$T = \begin{bmatrix} a & & \\ b & d & \\ c & e & f \end{bmatrix}$$

$$|a| = 1, \quad b = 0, \quad c = 0$$

$$|d| = 1, \quad e = 0, \quad |f| = 1$$

Inner Matrix

- U-sing zero-dimensional elements

$$T = \begin{bmatrix} \boxed{-} & & \\ c & -s & \\ s & c & | \end{bmatrix} \qquad c^2 + s^2 = 1$$

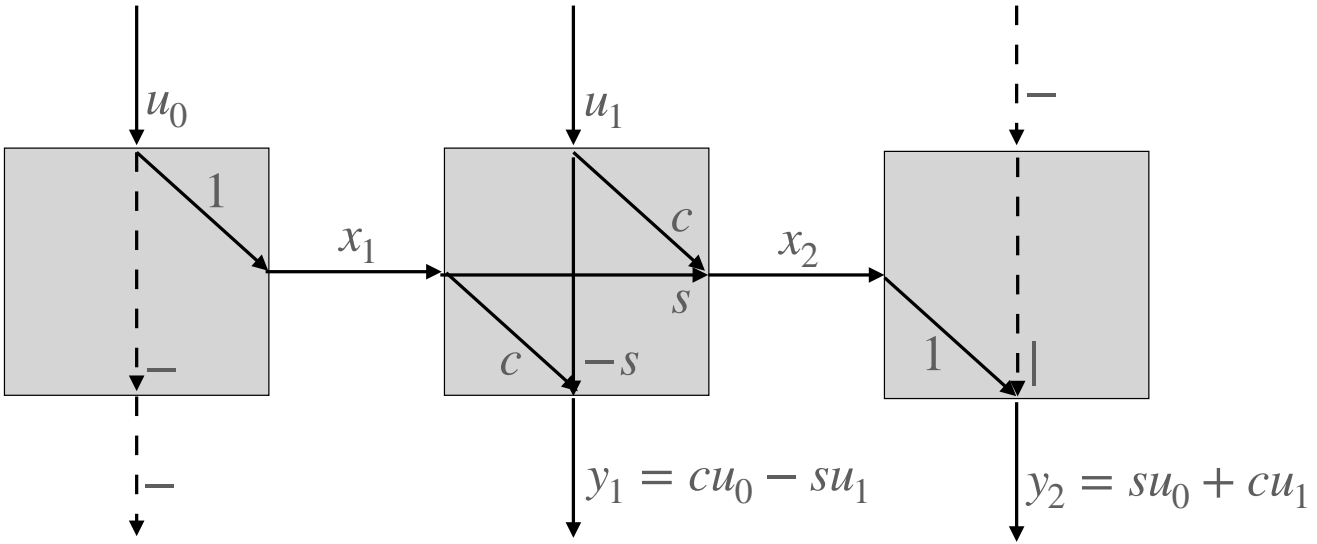
- Orthogonality

$$T^T T = \begin{bmatrix} | & c & s \\ & -s & c \\ & & - \end{bmatrix} \begin{bmatrix} - & & \\ c & -s & \\ s & c & | \end{bmatrix} = \begin{bmatrix} | - + c^2 + s^2 & sc - cs & s \, | \\ cs - sc & c^2 + s^2 & c \, | \\ - \, s & - \, c & - \, | \end{bmatrix} = \begin{bmatrix} 1 & 0 & | \\ 0 & 1 & | \\ - & - & \cdot \end{bmatrix}$$

Inner Matrix

$$T = \begin{bmatrix} \boxed{-} & & \\ c & -s & \\ s & c & | \end{bmatrix}$$

$$\left[\begin{array}{c|c} A_k & B_k \\ \hline C_k & D_k \end{array}\right] = \left[\begin{array}{c|c} | & 1 \\ \hline \cdot & - \end{array}\right]_{k=0} \quad \left[\begin{array}{c|c} s & c \\ \hline c & -s \end{array}\right]_{k=1} \quad \left[\begin{array}{c|c} - & \cdot \\ \hline 1 & | \end{array}\right]_{k=2}$$



Inner Matrix

- Tall matrix isometry

Error in Book

$$T = \begin{bmatrix} \boxed{s} & 0 \\ c^2 & s \\ -cs & c \end{bmatrix}$$

$$(s^2 + c^4 + c^2(1 - c^2) = s^2 + c^4 + c^2 - c^4 = 1)$$

$$T^T T = \begin{bmatrix} \boxed{s} & c^2 & -cs \\ 0 & s & c \end{bmatrix} \begin{bmatrix} \boxed{s} & 0 \\ c^2 & s \\ -cs & c \end{bmatrix} = \begin{bmatrix} s^2 + c^4 + c^2 s^2 & c^2 s - c^2 s \\ sc^2 - c^2 s & s^2 + c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$