Time-Varying Systems and Computations

Unit 4.3

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Standard QR Decomposition

- For a square, full rank matrix $T = QR$
- Orthonormal matrix $Q^T Q = Q Q^T = 1$
- Upper triangular matrix $R = \begin{bmatrix} * & * & * & \dots & * \\ & * & * & \dots & * \\ & & \ddots & & \vdots \\ & & & \ddots & \vdots \\ & & & & * \end{bmatrix}$

Alternative QR decompositions

- Standard QR Decomposition $T = QR$ row-wise elimination
- Transposed QRD $T = LQ$ column-wise elimination
- Flipped QRD $T = RQ$ column-wise elimination
- Flipped/Transposed QRD $T = QL$ row-wise elimination

Alternative QR decompositions

- The lower triangular factor allows interpretation as causal system
- Column-wise elimination corresponds to post-multiplication with Householder/Given/Generalized Rotations
- Compressing columns

$$T = \begin{bmatrix} 0 & R_2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

Alternative QR Schemes

- There exist many variations to the standard QR decomposition
- The variants differ in the eliminiation scheme
  - Row-wise elimination
  - Column-wise elimination
  - Compressing rows
  - Compressing columns
- Determine orthogonal basis for row-spaces or column-spaces