TECHNISCHE UNIVERSITÄT MÜNCHEN
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MASTER'S THESIS

Representing Controlled Transfer Operators for Convex Optimal Control

Problem description:

In engineering, and in particular, in robotics, we are frequently interested in solving a task in an energy-efficient way, be it the control of a chemical plant, making a drone maneuver to a target location, or a robotic hand grasp. Those kinds of tasks can be formulated as nonlinear optimal control problems. Unfortunately, those problems are generally non-convex, and solution procedures suffer from the curse of dimensionality. Operator-based representations for control have emerged as a promising direction to solve those control problems. The basic idea is to find convenient mathematical models of the dynamical system. The two most common ways to model a system are: 1. an internal state and a differential equation defining the geometry of a system state space (Poincaré, Lyapunov); 2. a linear systems response to a signal via the frequency domain (Wiener). As an alternative, operator-based representations have gained traction. They represent the solution operator of a dynamical system directly on a space of its observable functions, leading to so-called transfer or Koopman operators. This change of perspective allows one to frame nonlinear optimal control problems as a convex problem [3].

Method: Representations of transfer operators by their eigenfunctions in a setting with inputs, allowing for forecasting models with realistic theoretical settings [2, 1] and data-driven modeling from trajectory data of dynamical systems.

Goal: An extension of operator-based spectral representation to controlled systems and its use to transform nonlinear optimal control problems into convex optimization problems.

Outcome: A contribution to a control conference (CDC, ECC, ...).

Tasks:

- Literature research on operator-based modeling, related data-driven techniques and optimal control
- Derivation of data-driven representations from trajectory data
- Implementation of a proof of concept and its applicability to optimal control
- Evaluation

Bibliography:

[1] Petar Bevanda, Max Beier, Armin Lederer, Alexandre Capone, Stefan Georg Sosnowski, and Sandra Hirche. Gaussian Process-Based Representation Learning via Timeseries Symmetries. In *ICML 2024 Workshop on Geometry-grounded Representation Learning and Generative Modeling*, June 2024.

[2] Petar Bevanda, Max Beier, Armin Lederer, Stefan Sosnowski, Eyke Hüllermeier, and Sandra Hirche. Koopman Kernel Regression. In *37th Conference on Neural Information Processing Systems (NeurIPS 2023)*, 2023. arXiv:2305.16215 [cs, eess, math, stat].

[3] Petar Bevanda, Nicolas Hoischen, Stefan Sosnowski, Sandra Hirche, and Boris Houska. Data-Driven Optimal Feedback Laws via Kernel Mean Embeddings, July 2024. arXiv:2407.16407 [cs, eess, math, stat].

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