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F O R S C H U N G S P R A X I S

Integrators for Operator-Theoretic Optimal Control

Problem description:

Optimal control is a mathematical framework used to determine the best possible control strategy for a dynamic system over time. It involves finding a control policy that minimizes or maximizes a certain performance criterion, such as cost, energy, or time. Applications of optimal control are vast and include areas like aerospace engineering for trajectory optimization, economics for resource allocation, and robotics for path planning.
The Hamilton-Jacobi-Bellman (HJB) equation is used in optimal control to determine the optimal policy by solving a partial differential equation that describes the evolution of the value function over time. Solution schemes suffer from stability, convergence issues, and the curse of dimensionality.
Recent work treated the problem with *Convex operator-theoretic methods in stochastic control* [3]. With great success, statistical learning [4] was applied to the problem in [1]. From their analysis, one can derive the optimal value function as the weak solution to a nonlinear equation in the weights of the value function $V(x)$. Solving this equation efficiently is paramount for real-time application in fast dynamic systems like robotic control. To this end, integration methods exploiting the known operator A seem convenient. Exponential integrators [2] provide a way to take steps along the flow of the linear part, allowing for longer stepsizes and increased accuracy in the presence of stiffness.
This FP aims to understand and implement exponential integrators for the HJB equation and compare them to commonly used Euler schemes.

Work schedule:

- Literature research
- Derivation
- Implementation
- Evaluation

Bibliography:

[1] Petar Bevanda, Nicolas Hoischen, Stefan Sosnowski, Sandra Hirche, and Boris Houska. Data-Driven Optimal Feedback Laws via Kernel Mean Embeddings, July 2024. arXiv:2407.16407 [cs, eess, math, stat].
[2] Marlis Hochbruck and Alexander Ostermann. Exponential integrators. *Acta Numerica*, 19:209–286, May 2010.
[3] Boris Houska. Convex operator-theoretic methods in stochastic control, May 2023. arXiv:2305.17628 [math].
[4] Vladimir Kostic, Pietro Novelli, Andreas Maurer, Carlo Ciliberto, Lorenzo Rosasco, and Massimiliano Pontil. Learning Dynamical Systems via Koopman Operator Regression in Reproducing Kernel Hilbert Spaces. *Advances in Neural Information Processing Systems*, 35:4017–4031, December 2022.

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