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MASTER'S THESIS

Data-driven Semigroup Equivariant Neural Models

Problem description:

There has been growing interest in operator learning within the machine learning community due to its numerous applications in computer vision and AI for scientific purposes. While there are ongoing developments of generic methods to learn operators or their components [3], these approaches often overlook the inherent structure of the problem being addressed. Equivariance is a hugely successful (AlphaFold) concept that allows us to build structural properties into learning problems. Usually, models include equivariance to SO or SE groups (rotation and translation) to expand their operating range. Equivariance allows us to treat more general groups and build in their structure. Unfortunately, the corresponding group action needs to be known as a priori, or one needs to solve (ill-posed) inverse problems to uncover them. If observations with some underlying equivariant structure are available, it seems practicable to inform our learning method of this structure.

Method: For dynamical systems, an object of interest is the Semigroup that solves the associated differential equation [2]. We naturally observe solutions of these differential equations when learning from data, so we want to connect their solution operators to our model. By noticing the equivalence relation [1] (c.f. Fig. 1) in obtaining trajectories – F_t and solution operator A_t– we can inform our learning problem with a dynamical system prior.

Goal: The investigation and implementation of Semigroup methods for learning dynamical systems and engraining data-driven priors into the learning problem.

Outcome: A contribution to a machine-learning venue (ICML, Neurips, ICLR, TMLR, ...).

Tasks:

- Literature research
- Derivation
- Implementation
- Evaluation

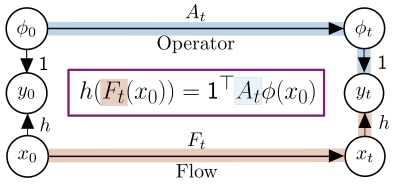

Figure 1: Equivariance relation

Bibliography:

[1] Petar Bevanda, Max Beier, Armin Lederer, Alexandre Capone, Stefan Georg Sosnowski, and Sandra Hirche. Gaussian Process-Based Representation Learning via Timeseries Symmetries. June 2024.

[2] Klaus-Jochen Engel and Rainer Nagel. *One-Parameter Semigroups for Linear Evolution Equations*, volume 194 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2000.

[3] J. Jon Ryu, Xiangxiang Xu, H. S. Melihcan Erol, Yuheng Bu, Lizhong Zheng, and Gregory W. Wornell. Operator SVD with Neural Networks via Nested Low-Rank Approximation, February 2024. arXiv:2402.03655 [cs, math, stat].

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