Set-Based Learning of Neural Barrier Certificates for Safety Verification of Dynamic Systems

Background

Safety-critical systems are usually modeled using dynamic systems. To ensure their safety, all components of the model must be formally verified. For example, given an initial set and an unsafe set, a verification goal is to show that no point from the initial set can reach the unsafe set given the dynamic of the system. One option to verify this setting is barrier certificates, validating the safety of continuous-time linear, nonlinear, and hybrid systems [3, 4].

A barrier certificate is a function mapping the state space into a scalar real-valued output for which three conditions must hold to guarantee safety: (i) The function output is negative for each point from the initial set, (ii) the function output is positive for each point from the unsafe set, and (iii) the dynamic of all points mapping to 0 guarantees that the points are pushed back into the negative region. These conditions ensure that one cannot reach the unsafe region starting from any point within the initial set under the given dynamic. Thus, the existence of such a function proves the safety of the system under consideration.

Recent approaches model this function using neural networks [5, 6], where the constraints of the barrier certificates are incorporated in the loss of the neural network. The network is then trained by sampling points from the initial set, the unsafe set, and the zero level set. However, these approaches suffer from three issues: (a) Sampling points becomes infeasible with high-dimensional systems resulting in weak scalability, (b) it is non-trivial how to obtain the zero level set, and (c) one has to use an external verifier to determine if the trained neural network is a valid barrier certificate.

Description

We aim to overcome these limitations using set-based learning [2] in the neural network training process. Firstly, The input set and unsafe set are no longer modeled by individual points but as continuous sets, for which set representations exist that scale well with the number of dimensions. Secondly, by propagating the considered domain through the network, one can extract the zero level set by exploiting the dependencies between the input and the output of the network. Finally, the loss function can be modeled such that the trained neural network embodies a valid barrier certificate as soon as the loss reaches 0, mitigating an additional verification step to validate the obtained barrier certificate.

Tasks

- Literature research of barrier certificates and set-based learning
- Familiarize with the toolbox CORA [1]
- Implementation of the neural network training to learn valid barrier certificates
- Refine the state space for more accurate computation, i.e., split the zero level set
- Evaluate the approach against state-of-the-art methods and benchmarks
- Optional: Adapt the size of the network dynamically to automatically adapt to harder problems

References

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