

Exploiting Monotonicity for Training and Verification of Neural Networks



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Background

Neural networks are great at solving many complex tasks, e.g., object detection [10], natural language processing [11], or chess [8]. To ensure the safety of neural networks in safety-critical environments, such as autonomous driving [12], there is a large field of research around the formal verification of neural networks [1]. However, the formal verification of neural networks is computationally hard [5]. Generally, there are two approaches to formally verify neural networks: The first approach encodes the specification and the neural networks as an optimization problem and applies an off-the-shelf solver, e.g., (mixed-integer) linear programming [3] or a satisfiability modulo theories solver [5]; The second approach computes a tight enclosure of the output set of a neural network, e.g., using interval bound propagation [4] or by propagating more expressive set representations like zonotopes [9, 7]. Recent research exploit monotonicity in neural networks to speed-up and improve the formal verification [6].

Description

For monotone functions, exact output bounds can be computed by propagating input bounds. Neural networks can be made monotone by small modification of their architecture; recent research shows promising results by making entire neural networks monotone [6]. However, such monotonic neural networks can only approximate monotone functions. In this thesis, we want to compose multiple monotonic neural networks to approximate arbitrary non-monotonic functions. The composition can be viewed as a mixed-monotone system [2]. Moreover, we want to investigate the performance of compositional monotonic neural networks and find efficient methods to exploit the mixed-monotonic behavior for their formal verification.

Tasks:

1. Literature research on formal verification of neural networks.
2. Implementation of monotonic neural network architecture and different methods for their composition.
3. Training different compositional monotonic neural networks and comparing the performance on monotonic and non-monotonic tasks.
4. Development of efficient algorithms for the formal verification of compositional monotonic neural networks.
5. Extensive evaluation and comparison with existing approaches for formal verification of neural networks.

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